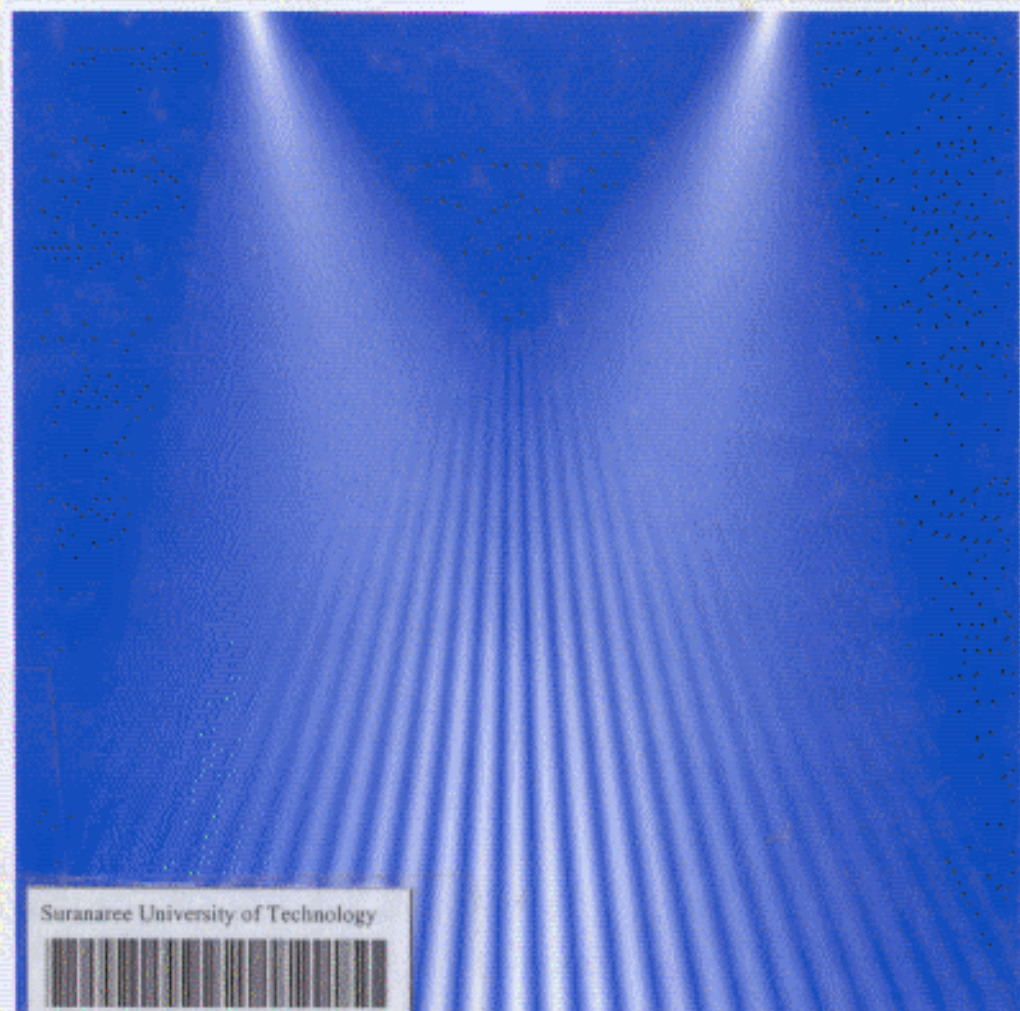


# A FIRST COURSE IN FOURIER ANALYSIS



Suranaree University of Technology



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