SECOND EDITION

## ELASTICITY IN ENGINEERING MECHANICS



ARTHUR P. BORESI

KEN P. CHONG

## **CONTENTS**

Preface			χV
CHAPTER 1	INTRO	DUCTORY CONCEPTS AND MATHEMATICS	1
Part I	Introd	uction	1
	1-1	Trends and Scopes	1
		Theory of Elasticity	4
		Numerical Stress Analysis	4
		General Solution of the Elasticity Problem	5
		Experimental Stress Analysis	6
	1-6	Boundary-Value Problems of Elasticity	6
Part II	Prelin	ninary Concepts	8
	1_7	Brief Summary of Vector Algebra	8
		Scalar Point Functions	12
,		Vector Fields	14
•		Differentiation of Vectors	15
		Differentiation of a Scalar Field	17
		Differentiation of a Vector Field	18
	1 14	TATE AT A TATA A	

÷ *	1-13	Curl of a Vector Field	19
	1-14	Eulerian Continuity Equation for Fluids	19
	1-15	Divergence Theorem	22
	1-16	Divergence Theorem in Two Dimensions	24
	1-17	Line and Surface Integrals (Application of	
		Scalar Product)	25
	-	Stokes's Theorem	27
	1-19	Exact Differential	27
		Orthogonal Curvilinear Coordinates in Three-Dimensional Space	28
		Expression for Differential Length in Orthogonal Curvilinear Coordinates	29
·		Gradient and Laplacian in Orthogonal Curvilinear Coordinates	30
Part III	Eleme	ents of Tensor Algebra	33
	1-23	Index Notation: Summation Convention	33
		Transformation of Tensors under Rotation of Rectangular Cartesian	25
		Coordinate System	37
		Symmetric and Antisymmetric Parts of a Tensor	44
	1-26	Symbols of $\delta_{ij}$ and $\epsilon_{ijk}$ (the Kronecker Delta and the Alternating Tensor)	45
	1-27	Homogeneous Quadratic Forms	47
	1-28	Elementary Matrix Algebra	50
•	1-29	Some Topics in the Calculus of Variations	55
	Refer	rences	59
	Biblio	ography	61
CHAPTER 2	THEC	DRY OF DEFORMATION	62
	2-1	Deformable, Continuous Media	62
	2-2	Rigid-Body Displacements	63
	2-3	Deformation of a Continuous Region. Material Variables. Spatial Variables	65
	2-4	Restrictions on Continuous Deformation	_
		of a Deformable Medium	69
		Problem Set 2-4	71
	2-5	Gradient of the Displacement Vector. Tensor Quantity	72

		CONTENTS	Ail
	2-6	Extension of an Infinitesimal Line	_,
	_	Element	74
		Problem Set 2-6	81
•		Physical Significance of $\epsilon_{ii}$ . Strain Definitions	82
•	2-8	Final Direction of Line Element.	
		Definition of Shearing Strain. Physical	85
		Significance of $\epsilon_{ij}(i \neq j)$	90
• .		Problem Set 2-8	90 91
	2-9	Tensor Character of $\epsilon_{\alpha\beta}$ . Strain Tensor	91
	2-10	Reciprocal Ellipsoid. Principal Strains. Strain Invariants	92
1	2-11	Determination of Principal Strains.	0.6
		Principal Axes	96
		Problem Set 2-11	102
	2-12	Determination of Strain Invariants.	104
		Volumetric Strain	104
	2-13	Rotation of a Volume Element.	109
		Relation to Displacement Gradients	113
		Problem Set 2-13	115
	2-14	Homogeneous Deformation	113
	2-15	Theory of Small Strains and Small	118
		Angles of Rotation	128
		Problem Set 2-15	120
	2-16	Compatibility Conditions of the Classical Theory of Small Displacements	130
	•	Problem Set 2-16	135
* .	0.15		
	2-17	7 Additional Conditions Imposed by Continuity	136
	2.19	8 Kinematics of Deformable Media	138
•	2-14	Problem Set 2-18	144
Appendix 2A	Stra	nin-Displacement Relations in Orthogonal	144
	Cur	vilinear Coordinates	
	2A-	-1 Geometrical Preliminaries	144
		-2 Strain-Displacement Relations	146
Appendix 2B	Dei for	rivation of Strain–Displacement Relations Special Coordinates by Cartesian Methods	150

Appendix 2C		n–Displacement Relations in General dinates	153
•	2C-1	Euclidean Metric Tensor	153
	2C-2	Strain Tensors	156
	Refe	rences	157
	Bibli	ography	158
CHAPTER 3	THE	ORY OF STRESS	159
	3-1	Definition of Stress	159
	3-2		162
	3-3	Summation of Moments. Stress at a Point. Stress on an Oblique Plane	164
		Problem Set 3-3	169
	2.4		107
	3-4	Transformation of Stress Components	
		under Rotation of Coordinate Axes	173
		Problem Set 3-4	176
	3-5		
	5 0	Extreme Values	177
		Problem Set 3-5	181
	3-6	Mean and Deviator Stress Tensors.	182
		Octahedral Stress	187
	2.7	Problem Set 3-6	107
	3-7	Approximations of Plane Stress. Mohr's Circles in Two and Three Dimensions	191
		Problem Set 3-7	198
	3-8	Differential Equations of Motion of a	
		Deformable Body Relative to Spatial Coordinates	199
•		Problem Set 3-8	203
Annondiv 2A	Diff	erential Equations of Equilibrium In	
Appendix 3A		villnear Spatial Coordinates	204
	3A-	1 Differential Equations of Equilibrium in Orthogonal Curvilinear Spatial Coordinates	204
	3A-	2 Specialization of Equations of	
		Equilibrium	206
	3A-	3 Differential Equations of Equilibrium in General Spatial Coordinates	208

CONTENTS	X
----------	---

Appendix 3B	Equations of Equilibrium Including Couple Stress and Body Couple		
	Quess.	and Body Couple	210
Appendix 3C	Reduction Sm	tion of Differential Equations of Motion all-Displacement Theory	212
		Material Derivative. Material Derivative of a Volume Integral	212
	3C-2	Differential Equations of Equilibrium Relative to Material Coordinates	216
		References	222
		Bibliography	223
CHAPTER 4		E-DIMENSIONAL EQUATIONS OF TICITY	224
		Elastic and Nonelastic Response of a Solid	224
	4-2	Intrinsic Energy Density Function (Adiabatic Process)	228
	4-3	Relation of Stress Components to Strain	230
	4.4	Energy Density Function Generalized Hooke's Law	233
:	4-4	Problem Set 4-4	243
	15	Isotropic Media. Homogeneous Media	243
	4-5 4-6	Strain Energy Density for Elastic	
	4-0	Isotropic Medium	244
		Problem Set 4-6	250
	4-7	Special States of Stress	254
· ·		Problem Set 4-7	257
	4-8	Equations of Thermoelasticity	257
	4-9	Differential Equation of Heat Conduction	259
		Elementary Approach to Thermal-Stress Problem in One and Two Variables	261
	4-11	Stress-Strain-Temperature Relations	265
		Problem Set 4-11	272
	4-12	Thermoelastic Equations in Terms of Displacement	274

	4-13	Spherically Symmetrical Stress Distribution (The Sphere)	277
		Problem Set 4-13	279
	4-14	Thermoelastic Compatibility Equations in Terms of Components of Stress and	
		Temperature. Beltrami-Michell Relations	279
		Problem Set 4-14	284
	4-15	<b>Boundary Conditions</b>	286
		Problem Set 4-15	290
	4-16	Uniqueness Theorem for Equilibrium Problem of Elasticity	290
	4-17	Equations of Elasticity in Terms of	
		Displacement Components	295
		Problem Set 4-17	297
	4-18	Elementary Three-Dimensional Problems of Elasticity. Semi-Inverse Method	298
		Problem Set 4-18	304
	4-19	Torsion of Shaft with Constant Circular Cross Section	308
		Problem Set 4-19	312
	4-20	Energy Principles in Elasticity	314
		Principle of Virtual Work	315
		Problem Set 4-21	320
	4-22	Principle of Virtual Stress (Castigliano's Theorem)	321
	4-23	Mixed Virtual Stress-Virtual Strain	
		Principles (Reissner's Theorem)	323
Appendix 4A		ication of the Principle of Virtual Work	
		Deformable Medium (Navier–Stokes	324
	Equa	itions)	324
Appendix 4B	Nonli	inear Constitutive Relationships	327
	4B-1	Variable Stress-Strain Coefficients	328
	4B-2	Higher-Order Relations	328
		Hypoelastic Formulations	328
		Summary	329
	Refe	rences	329
	Bibli	ography	332

CONTENTS	

χi

CHAPTER 5	PLAN RECT	IE THEORY OF ELASTICITY IN ANGULAR CARTESIAN COORDINATES	333
	5-1	Plane Strain	334
	-	Problem Set 5-1	338
	5-2	Generalized Plane Stress	340
	· · · ·	Problem Set 5-2	344
	5-3	Compatability Equation in Terms of	246
		Stress Components	346
		Problem Set 5-3	350
	5-4	_ · · •	351
		Problem Set 5-4	361
	5-5		368
		Harmonic Functions	300
•	5-6		369
		Elasticity Problem Set 5-6	373
•	57		5.5
	5-7	Dimensional Problems in Rectangular	
		Cartesian Coordinates	377
-		Problem Set 5-7	380
	5-8		201
		Displacement Components	384
		Problem Set 5-8	385
	5-9		206
		Coordinate Axes	386
Appendix 5A	Plar	ne Elasticity with Couple Stresses	390
	5A-	1 Introduction	390
	5A-	2 Equations of Equilibrium	391
	5A-	3 Deformation in Couple-Stress Theory	391
	5A·	4 Equations of Compatibility	393
	5A-	-5 Stress Functions for Plane Problems with	206
		Couple Stresses	396
Appendix 5B		ne Theory of Elasticity in Terms of mplex Variables	398
•	5B	-1 Airy Stress Function in Terms of Analytic Functions $\psi(z)$ and $\chi(z)$	398

	5B-2	Displacement Components in Terms of Analytic Functions, $\psi(z)$ and $\chi(z)$	399
	5B-3	Stress Components in Terms of $\psi(z)$ and $\chi(z)$	400
	5B-4	Expressions for Resultant Force and	700
	J <b>D</b> .	Resultant Moment	403
	5B-5	Mathematical Form of Functions $\psi(z)$ and $\chi(z)$	404
	5B-6	Plane Elasticity Boundary-Value Problems in Complex Form	408
	5B-7	Note on Conformal Transformation	411
		Plane Elasticity Formulas in Terms of Curvilinear Coordinates	416
	5B-9	Complex Variable Solution for Plane Region Bounded by Circle in the z Plane	419
		Problem Set 5B	423
	Refe	rences	424
		iography .	425
		- S-wp-1	120
CHAPTER 6	PLAI	NE ELASTICITY IN POLAR COORDINATES	427
	6-1	Equilibrium Equations in Polar Coordinates	427
•	6-2	Stress Components in Terms of Airy Stress Function $F = F(r, \theta)$	428
	6-3	Strain-Displacement Relations in Polar	
		Coordinates	430
		Problem Set 6-3	432
	6-4	Stress-Strain-Temperature Relations	433
		Problem Set 6-4	435
	6-5	Compatibility Equation for Plane	40.5
		Elasticity in Terms of Polar Coordinates	435
		Problem Set 6-5	436
	6-6		438
	6-7	Problem Set 6-6	449
	0-/	Plane-Elasticity Equations in Terms of Displacement Components	451
	6-8	Plane Theory of Thermoelasticity	455
•	O-O	Problem Set 6-8	458
		Disk of Variable Thickness and	750
	6-9	THIS OF VARIABLE INICKBESS AND	

		CONTENTS	xiii
	6-10	Problem Set 6-9 Stress Concentration Problem of Circular	465
		Hole in Plate	465
		Problem Set 6-10	472
		Examples	473
	Prob	lem Set 6-11	478
Appendix 6A		s-Couple Theory of Stress Concentration liting from Circular Hole in Plate	487
Appendix 6B		s Distribution of a Diametrically pressed Plane Disk	492
Dec.	Refe	rences	494
CHAPTER 7	PRIS	MATIC BAR SUBJECTED TO END LOAD	496
	7-1	General Problem of Three-Dimensional Elastic Bars Subjected to Transverse End Loads	496
	7-2	Solution. Warping Function	499
		Problem Set 7-2	505
	7-3		505
		Problem Set 7-3	509
	7-4	11 1/10/11/04 01 001/11/11/11 01 //	£10
		Problem: Elliptic Cross Section	510 514
	7.5	Problem Set 7-4	314
	7-5	Remarks on Solutions of the Laplace Equation, $\nabla^2 F = 0$	515
		Problem Set 7-5	517
4.	7-6	Torsion of Bars with Tubular Cavities	520
	, 0	Problem Set 7-6	523
	7-7		523
	7-8		
		Direction	524
		Problem Set 7-8	529
	7-9	Solution of Torsion Problem by the	
		Prandtl Membrane Analogy	529
		Problem Set 7-9	538

	7-10	Solution by Method of Series.	
		Rectangular Section	531
		Problem Set 7-10	543
	7-11	* Bending of a Bar Subjected to Transverse	
		End Force	544
;		Problem Set 7-11	550
	7-12		
. *		Subjected to Transverse End Force	550
	7 12	Problem Set 7-12	559
	/-13	Center of Shear Problem Set 7-13	560
	7 14		56
	/-14	Bending of a Bar with Elliptic Cross Section	563
	7-15	Bending of a Bar with Rectangular	50.
	7-10	Cross Section	565
		Problems Set 7-15	570
	÷ .	Review Problems	571
			0,,
Appendix 7A	Anal	ysis of Tapered Beams	572
	Refe	erences	576
01115TTD 0			
CHAPTER 8	GEN	ERAL SOLUTIONS OF ELASTICITY	578
٠,	8-1	Introduction	578
		Problem Set 8-1	579
	8-2	Equilibrium Equations	579
		Problem Set 8-2	581
	8-3	The Helmholtz Transformation	581
		Problem Set 8-3	583
	8-4	The Galerkin (Papkovich) Vector	583
		Problem Set 8-4	585
	8-5	Stress in Terms of the Galerkin Vector F	585
		Problem Set 8-5	586
	8-6	The Galerkin Vector: A Solution of the	<b>~</b> ^
		Equilibrium Equations of Elasticity	586
	07	Problem Set 8-6 The Colorlein Vector 1-7 and Leave's Station	588
	0-/	The Galerkin Vector kZ and Love's Strain Function for Solids of Revolution	588
		Problem Set 8-7	501

	CONTENTS	AV
8-8	Kelvin's Problem: Single Force Applied in the Interior of an Infinitely Extended	
	Solid	591
	Problem Set 8-8	593
8-9	The Twinned Gradient and Its	
	Application to Determine the Effects of	
	a Change of Poisson's Ratio	593
8-10	Solutions of the Boussinesq and	
	Cerruti Problems by the Twinned Gradient	
	Method	597
	Problem Set 8-10	600
8-11	Additional Remarks on Three-	
	Dimensional Stress Functions	600
References		601
Bibliography		601
INDEX		603

٠

CONTENTO