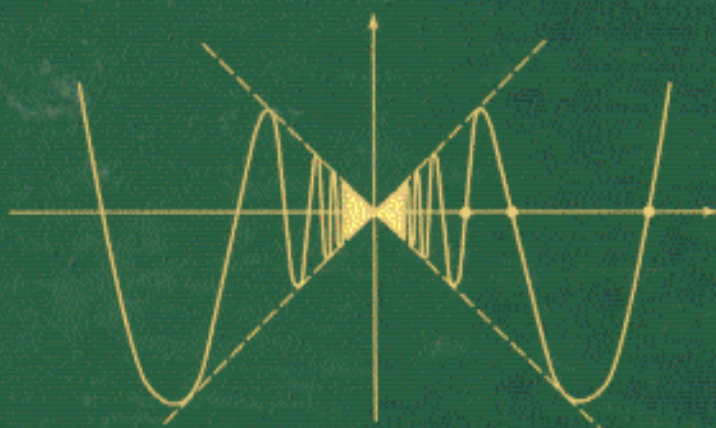


ROBERT G. BARTLE  
DONALD R. SHERBERT

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INTRODUCTION TO  
REAL  
ANALYSIS

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THIRD EDITION

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