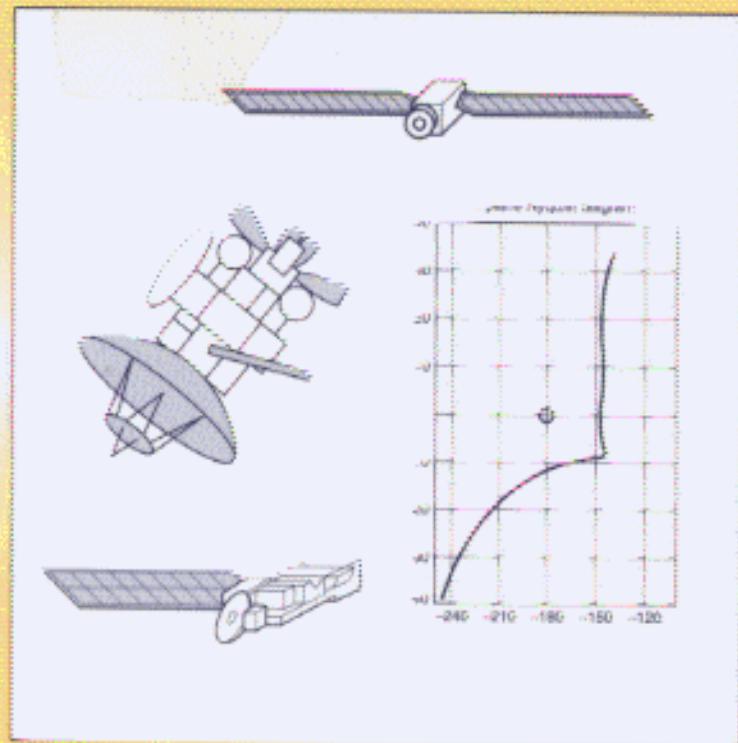


# **CLASSICAL FEEDBACK CONTROL**

With MATLAB



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