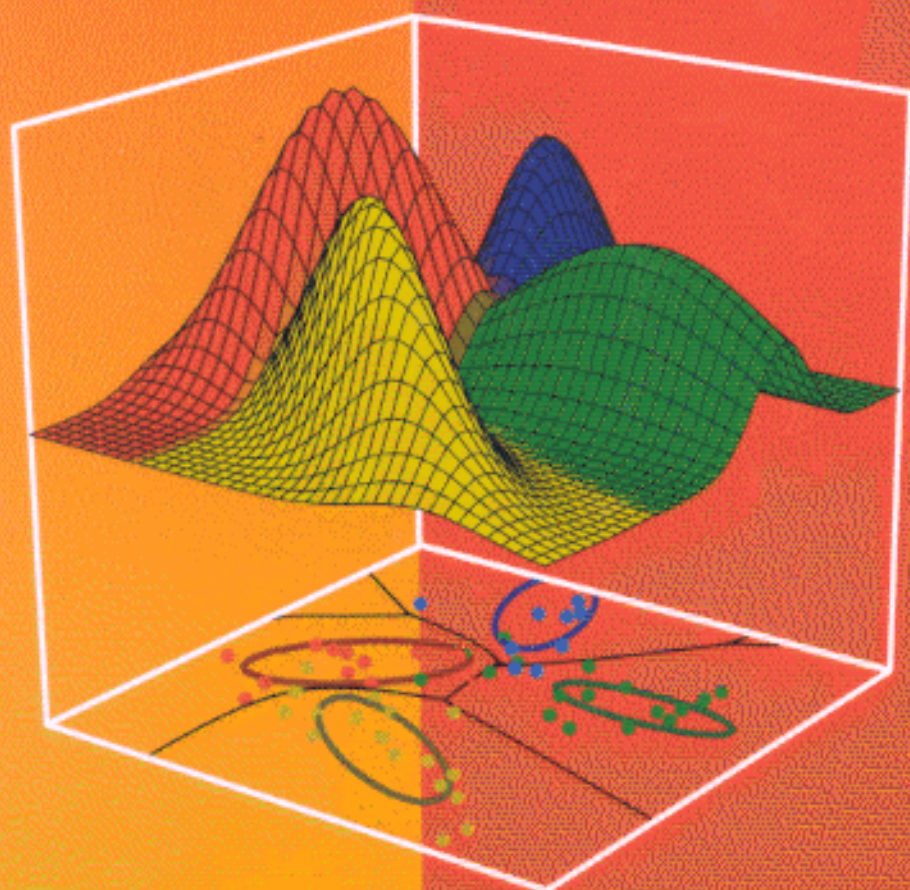


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Pattern Classification



Second Edition

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