

**HANDBOOK OF
MATHEMATICAL
FORMULAS**
2ND EDITION
AND INTEGRALS

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Preface to the Second Edition xxi

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