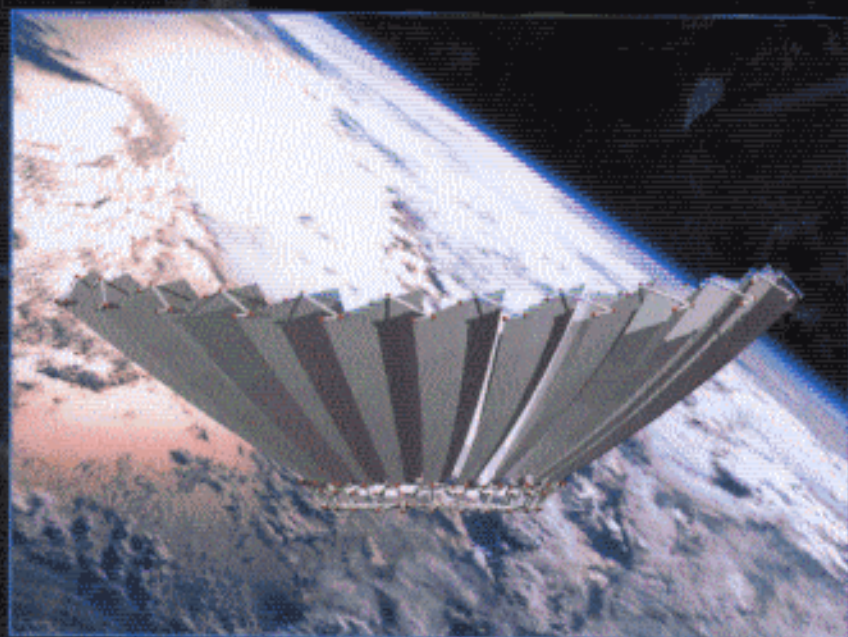


 WILEY

Flexible Multibody Dynamics

A Finite Element Approach



Michel Géradin
Alberto Cardona

Contents

Preface	xi
1 Introduction	1
1.1 The concept of a flexible multibody system	1
1.2 Contents	6
2 Generalized Coordinates for Mechanism Analysis	9
2.1 The four-bar mechanism example	10
2.2 Description in terms of minimal coordinates	11
2.3 Description in terms of Lagrangian coordinates	13
2.3.1 Hartenberg-Denavit method	14
2.4 Description in terms of Cartesian coordinates	17
2.5 Description in terms of finite element coordinates	18
2.5.1 Strong form of kinematic constraints for kinematic analysis	18
2.5.2 The zero strain energy approach	20
2.5.3 Numerical solution of the kinematic problem	21
2.5.4 Generalization to statics	23
2.5.5 Generalization to dynamics	24
2.6 Example: kinematics of deployment of solar array	26
2.7 Time integration of equations of motion	29
2.8 Example: the double pendulum	33
3 Kinematics of Finite Motion	39
3.1 Matrix representation of vector operations	40
3.2 Kinematic description of rigid body motion	44
3.2.1 Spherical motion	44
3.2.2 Non-commutative character of finite rotations	48
3.2.3 Explicit expressions of the rotation operator	49
3.2.4 General motion of a rigid body	52
3.3 Velocity analysis of rigid motion	55
3.3.1 Velocity analysis of spherical motion	55

3.3.2	Explicit expression of the angular velocities	56
3.3.3	Velocity analysis of arbitrary body motion. Instantaneous screw axis	58
3.4	Acceleration analysis of rigid motion	60
3.4.1	Acceleration analysis of spherical motion	60
3.4.2	Explicit expression of angular accelerations	61
3.4.3	Time rate of change of instantaneous rotation axis	61
3.5	Infinitesimal spherical motion and rotation increments	62
3.5.1	Spatial and material infinitesimal rotations	62
3.5.2	Variation of angular velocities	63
3.5.3	Angular velocities and accelerations in a moving frame	64
3.5.4	Incremental rotations as unknowns	65
4	Parameterization of Spherical Motion	67
4.1	Parameterization of rigid body spherical motion	69
4.2	The Cartesian rotation vector	69
4.3	Cayley form of rotation matrix – Rodrigues parameters	71
4.4	Finite rotations in terms of Euler parameters	73
4.5	Quaternion algebra and finite rotations	76
4.6	The Conformal Rotation Vector (CRV)	81
4.7	The linear parameters	83
4.8	Geometric description of finite rotations	84
4.8.1	Euler angles	84
4.8.2	Bryant angles	86
5	Rigid Body Dynamics	89
5.1	Kinematic description	89
5.2	Kinetic energy	90
5.3	Potential energy	92
5.4	Equations of motion in standard form	93
5.5	Equations of motion in parameterized form	95
5.6	Incremental form of the motion equations	97
5.7	Exact linearization at equilibrium	100
5.8	Example: top motion in a gravity field	101
6	The Elastic Beam	105
6.1	Beam kinematics	107
6.2	The displacement gradient measure of deformation	108
6.3	Pseudo-polar decomposition of the Jacobian matrix	110
6.4	The Green strain tensor	111
6.5	Local form of equilibrium	113
6.6	Variation of beam strains	117
6.6.1	Weak form of beam equations	118
6.6.2	Constitutive law	119

6.7	Displacement finite element modelling	120
6.7.1	Discretization	120
6.7.2	Construction of the beam strain matrix	120
6.7.3	Discretized form of the dynamic equilibrium equations	121
6.7.4	Linearization of dynamic equilibrium equations	122
6.8	Shear locking and reduced integration	127
6.9	Examples	131
6.9.1	Cantilever beam: effect of residual bending flexibility correction	131
6.9.2	Cantilever beam with two transverse loads	132
6.9.3	Cantilever 45-degree bend	133
6.9.4	Lee frame	134
6.9.5	Clamped-hinged circular arch	135
6.9.6	Out of plane buckling of a right-angle frame	136
7	System Constraints: Modelling of Joints	139
7.1	Types of constraints encountered in kinematic analysis	140
7.2	Numerical solution of constrained algebraic problems	142
7.2.1	The constraint elimination method	143
7.2.2	The Lagrange multiplier method	144
7.2.3	The penalty function method	144
7.2.4	The augmented Lagrangian method	145
7.2.5	The perturbed Lagrangian method	146
7.3	Unconstrained dynamic problems	146
7.4	Constrained dynamic problems	147
7.4.1	The case of holonomic constraints	147
7.4.2	The case of nonholonomic constraints	149
7.5	Classification of kinematic pairs	150
7.5.1	Lower pairs	150
7.5.2	Higher pairs	152
7.6	Modelling of lower-pair joints	152
7.6.1	Formulation of the hinge joint	153
7.6.2	The prismatic joint	155
7.7	Other infinitely rigid lower pairs	157
7.7.1	The cylindrical joint	157
7.7.2	The screw joint	158
7.7.3	The planar joint	159
7.7.4	The spherical joint	160
7.7.5	The rigid connection	160
7.8	Modelling of some higher-pair joints	160
7.8.1	The universal joint	160
7.8.2	The point-to-plane joint	161
7.8.3	The curvilinear slider	162
7.8.4	The rigid wheel	163

7.8.5	The rolling coin	166
7.9	Flexible effects in joints	167
7.9.1	Flexible hinge joint	167
7.9.2	Bushing connector	169
7.9.3	Flexible wheel	171
7.10	Interference between links	174
7.11	Example: retraction of a three-longeron truss	176
7.12	Conclusion	182
8	Substructuring Techniques	185
8.1	Concept of mechanical impedance	187
8.2	The Craig-Bampton method	191
8.3	Mechanical admittance for discrete systems	192
8.4	Restricted admittance: Mac Neal and Rubin methods	195
8.4.1	Mac Neal's method	197
8.4.2	Rubin's method	197
8.5	Nonlinear description of a superelement	198
8.5.1	Computation of the weight coefficients α	202
8.6	Computation of the strain energy	203
8.7	Corotational evaluation of the kinetic energy	204
8.7.1	Variation of kinetic energy and inertia forces	206
8.7.2	Tangent mass and pseudo damping matrices	207
8.8	Examples	208
8.8.1	Hinged beam	208
8.8.2	Beam on a spherical joint	212
8.8.3	Deployment of the MEA antenna	213
9	Static and Kinematic Analyses of Multibody Systems	219
9.1	Continuation methods	221
9.2	Tracing the equilibrium path in structural analysis	222
9.3	Selection of an appropriate metric	225
9.4	Scheme to advance the solution	227
9.4.1	Predictor step	227
9.4.2	Corrector step	229
9.5	Remarks on implementation aspects	231
9.6	Formulation of flexible mechanisms problems	233
9.7	Numerical applications	234
9.7.1	Three pinned-bar structure	235
9.7.2	Two arms mechanism	236
9.7.3	Thermal buckling problem	238
9.7.4	Buckling of a two hinged beam structure with axial load	239
9.8	Singular points detection along the equilibrium path	241
9.8.1	Solution of the system of equations	243
9.9	Terms involving derivatives of the tangent matrix	244

9.10	Null eigenvector updating under change of reference	245
9.10.1	Null eigenvector updating in beam models	247
9.11	Computation of the path tangents at a singular point	248
9.12	Numerical applications	250
9.12.1	Rigid bar/springs mechanism	250
9.12.2	Clamped beam	252
9.12.3	Right-angle frame	254
9.12.4	Deep circular arch under vertical load	257
9.13	Conclusions	259
10	Time Integration of Constrained Systems	263
10.1	Solution of dynamic constrained systems	265
10.1.1	Constraint regularization	266
10.1.2	Constraint reduction	266
10.1.3	Newmark's method	267
10.2	Equations of motion of constrained dynamic systems	269
10.3	Eigenvalue analysis	271
10.4	Stability analysis	275
10.5	Stability of time integration methods for DAE systems	277
10.5.1	The Newmark method without numerical dissipation	279
10.5.2	The Newmark method with numerical dissipation	280
10.5.3	The Hilber-Hughes-Taylor algorithm	281
10.5.4	The Generalized- α method	281
10.6	Example: the double pendulum	281
11	Automatic Step Size Control	285
11.1	Local truncation error estimation	286
11.2	Local error analysis for the SDOF oscillator	287
11.2.1	Local integration error	287
11.2.2	Expected value of the non-dimensional error	288
11.2.3	Time integration strategy	289
11.3	Local error analysis of uncoupled MDOF systems	289
11.3.1	Local integration error	289
11.3.2	Expected value of the non-dimensional error	290
11.3.3	Time integration strategy	290
11.4	Local error analysis of coupled MDOF systems	292
11.4.1	Projection on a modal basis	292
11.4.2	Bounds on the modal displacement amplitudes	292
11.4.3	Time integration strategy	293
11.5	Strategy for changing the time step	295
11.6	Numerical examples	296
11.6.1	1-DOF linear oscillator	296
11.6.2	Articulated beams with locking mechanism	297
11.6.3	Double pendulum with impulsive behaviour	299

11.7 Concluding remarks	300
12 Energy Conserving Time Integration	303
12.1 General formulation of a multibody dynamics problem	304
12.2 Time discretization by the mid-point rule	306
12.3 Energy conservation	307
12.4 Application to top motion	308
12.4.1 Rotation parameterization	309
12.4.2 Numerical behaviour	311
12.5 The case of elastic systems	312
12.5.1 Energy conservation and internal force averaging	315
12.6 Conclusion	316
References	317
Index	325