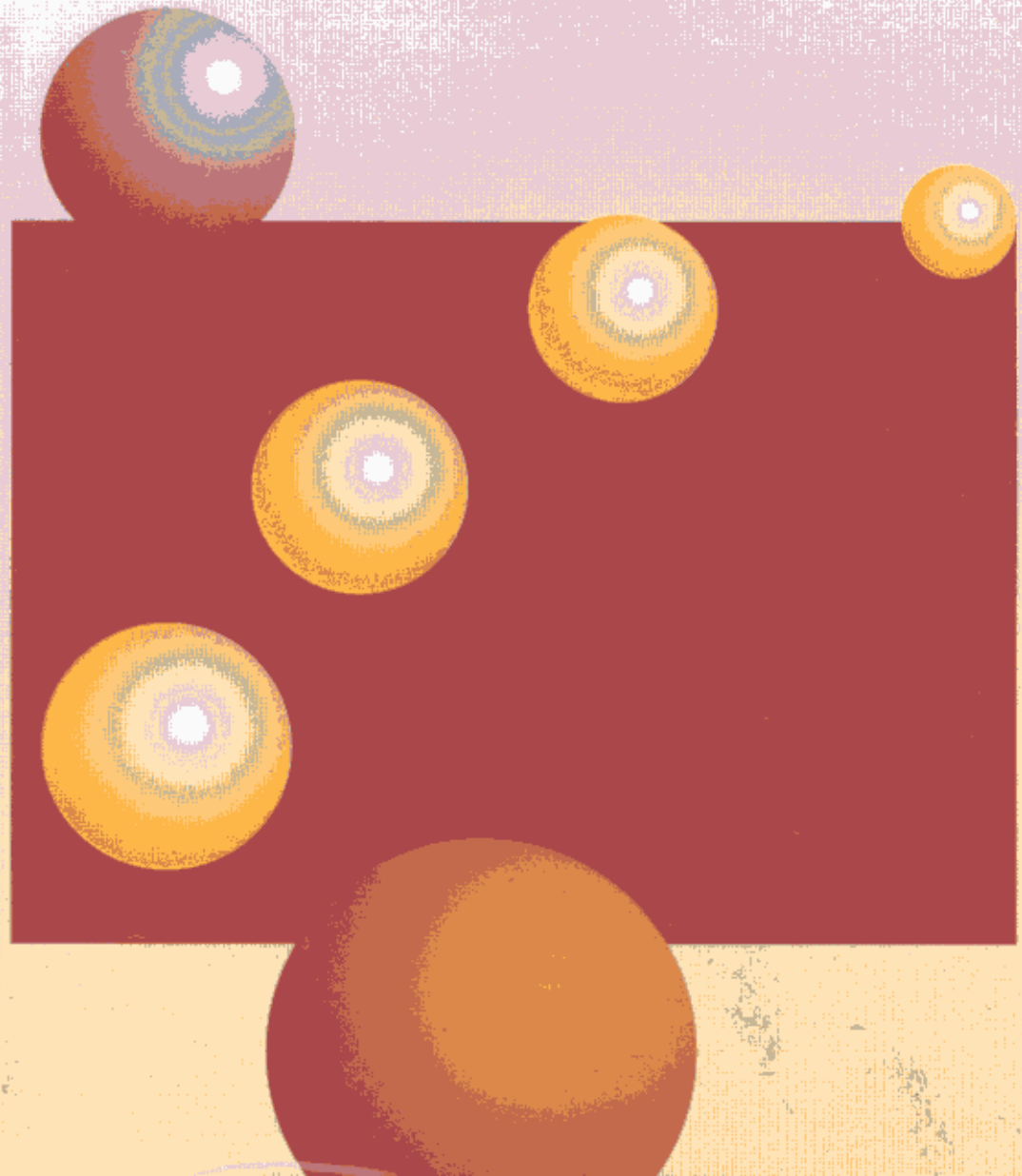
AMITABHA LAHIRI • PALASH B. PAL

A FIRST BOOK OF QUANTUM FIELD THEORY



Contents

Preface *v*

Notations *ix*

1 Preliminaries **1**
1.1 Why Quantum Field Theory 1
1.2 Creation and annihilation operators 3
1.3 Special relativity 5
1.4 Space and time in relativistic quantum theory 8
1.5 Natural units 9

2 Classical Field Theory **11**
2.1 A quick review of particle mechanics 11
2.1.1 Action principle and Euler-Lagrange equations . . . 11
2.1.2 Hamiltonian formalism and Poisson brackets 13
2.2 Euler-Lagrange equations in field theory 14
2.2.1 Action functional and Lagrangian 14
2.2.2 Euler-Lagrange equations 16
2.3 Hamiltonian formalism 17
2.4 Noether's theorem 20

3 Quantization of scalar fields **27**
3.1 Equation of motion 27
3.2 The field and its canonical quantization 28
3.3 Fourier decomposition of the field 29
3.4 Ground state of the Hamiltonian and normal ordering . . . 33
3.5 Fock space 35
3.6 Complex scalar field 36
3.6.1 Creation and annihilation operators 36
3.6.2 Particles and antiparticles 38
3.6.3 Ground state and Hamiltonian 39
3.7 Propagator 40

4 Quantization of Dirac fields 46
4.1 Dirac Hamiltonian . . . 46
4.2 Dirac equation . . . 50
4.3 Plane wave solutions of Dirac equation . . . 53
4.3.1 Positive and negative energy spinors . . . 53
4.3.2 Explicit solutions in Dirac-Pauli representation . . . 55
4.4 Projection operators . . . 57
4.4.1 Projection operators for positive and negative energy states . . . 57
4.4.2 Helicity projection operators . . . 58
4.4.3 Chirality projection operators . . . 60
4.4.4 Spin projection operators . . . 61
4.5 Lagrangian for a Dirac field . . . 61
4.6 Fourier decomposition of the field . . . 63
4.7 Propagator . . . 66

5 The S-matrix expansion 68
5.1 Examples of interactions . . . 69
5.2 Evolution operator . . . 71
5.3 S-matrix . . . 76
5.4 Wick's theorem . . . 78

6 From Wick expansion to Feynman diagrams 83
6.1 Yukawa interaction : decay of a scalar . . . 83
6.2 Normalized states . . . 90
6.3 Sample calculation of a matrix element . . . 93
6.4 Another example: fermion scattering . . . 97
6.5 Feynman amplitude . . . 101
6.6 Feynman rules . . . 102
6.7 Virtual particles . . . 106
6.8 Amplitudes which are not S-matrix elements . . . 108

7 Cross sections and decay rates 111
7.1 Decay rate . . . 111
7.2 Examples of decay rate calculation . . . 113
7.2.1 Decay of a scalar into a fermion-antifermion pair . . . 113
7.2.2 Muon decay with 4-fermion interaction . . . 119
7.3 Scattering cross section . . . 126
7.4 Generalities of 2-to-2 scattering . . . 129
7.4.1 CM frame . . . 131
7.4.2 Lab frame . . . 133
7.5 Inelastic scattering with 4-fermion interaction . . . 136
7.5.1 Cross-section in CM frame . . . 138
7.5.2 Cross-section in Lab frame . . . 139

7.6 Mandelstam variables 140

8 Quantisation of the electromagnetic field **142**
8.1 Classical theory of electromagnetic fields 142
8.2 Problems with quantization 145
8.3 Modifying the classical Lagrangian 146
8.4 Propagator 149
8.5 Fourier decomposition of the field 152
8.6 Physical states 154
8.7 Another look at the propagator 158
8.8 Feynman rules for photons 160

9 Quantum electrodynamics **162**
9.1 Local gauge invariance 162
9.2 Interaction Hamiltonian 166
9.3 Lowest order processes 168
9.4 Electron-electron scattering 170
9.5 Electron-positron scattering 176
9.6 $e^-e^+ \to \mu^-\mu^+$ 178
9.7 Consequence of gauge invariance 180
9.8 Compton scattering 182
9.9 Scattering by an external field 190
9.10 Bremsstrahlung 194

10 P, T, C and their combinations **196**
10.1 Motivations from classical physics 196
10.2 Parity 197
10.2.1 Free scalar fields 197
10.2.2 Free Dirac field 198
10.2.3 Free photon field 200
10.2.4 Interacting fields 201
10.3 Charge conjugation 203
10.3.1 Free fields 203
10.3.2 Interactions 206
10.4 Time reversal 208
10.4.1 Antilinearity 208
10.4.2 Free fields 209
10.4.3 Interactions 212
10.5 CP 213
10.6 CPT 214

11 Electromagnetic form factors **217**
11.1 General electromagnetic vertex 217
11.2 Physical interpretation of form factors 219
11.2.1 Charge form factor F_1 219

11.2.2 Anomalous magnetic moment F_2 222
11.2.3 Electric dipole moment $\widetilde{F}_2$ 223
11.2.4 Anapole moment $\widetilde{F}_3$ 224
11.3 Anomalous magnetic moment of the electron 225
11.4 Charge form factor 234
11.5 Electron-proton scattering 236

12 Renormalisation **240**
12.1 Degree of divergence of a diagram 240
12.1.1 Superficial degree of divergence 240
12.1.2 Superficial vs. real degree of divergence 243
12.2 Specific examples in QED 244
12.3 Outline of the program 246
12.4 Ward-Takahashi identity 247
12.5 General forms for divergent amplitudes 250
12.5.1 Fermion self-energy 250
12.5.2 Vacuum polarization 252
12.5.3 Vertex function 254
12.6 Regularization of self-energy diagrams 254
12.6.1 Vacuum polarization diagram 254
12.6.2 Fermion self-energy diagram 259
12.7 Counterterms 261
12.7.1 Vacuum polarization diagram 261
12.7.2 Fermion self-energy diagram 264
12.7.3 Vertex function 266
12.8 Full Lagrangian 267
12.9 Observable effects of renormalization 269
12.9.1 Modification of Coulomb interaction 269
12.9.2 Running coupling constant 270
12.9.3 Cancellation of infra-red divergences 273

13 Symmetries and symmetry breaking **277**
13.1 Classification of symmetries 277
13.2 Groups and symmetries 278
13.2.1 Symmetry group 278
13.2.2 Examples of continuous symmetry groups 280
13.2.3 Generators of continuous groups 284
13.2.4 Representations 286
13.3 Approximate symmetries 287
13.4 Spontaneous breaking of symmetries 288
13.4.1 Discrete symmetry 289
13.4.2 U(1) symmetry 293
13.4.3 Non-Abelian symmetry 294
13.5 Goldstone's theorem 295

13.5.1 Appearance of massless states 295
13.5.2 Examples of Nambu-Goldstone bosons 297
13.5.3 Interaction of Goldstone bosons 299
13.6 Higgs mechanism 302

14 Yang-Mills theory of non-Abelian gauge fields **306**
14.1 Gauge fields of non-Abelian symmetry 306
14.2 Pure gauge Lagrangian 309
14.3 Interactions of non-Abelian gauge fields 312
14.3.1 Gauge interactions of other particles 312
14.3.2 Self-interactions of gauge bosons 313
14.4 Equations of motion and conserved currents 315
14.5 Quantization of non-Abelian gauge fields 316
14.6 Quantum Chromodynamics 317

15 Standard electroweak theory **320**
15.1 Gauge group 320
15.1.1 Choice of gauge group 320
15.1.2 Pure gauge Lagrangian 322
15.2 Spontaneous symmetry breaking 324
15.2.1 Introducing the Higgs boson multiplet 324
15.2.2 Gauge boson masses 326
15.2.3 Scalar modes 327
15.3 Fermions in the theory 329
15.3.1 Gauge interactions 329
15.3.2 Electron mass 332
15.3.3 Yukawa couplings 333
15.3.4 Other fermions in the model 334
15.4 Gauge boson decay 336
15.5 Scattering processes 339
15.5.1 Forward-backward asymmetry 339
15.5.2 Low energy weak interactions 342
15.5.3 High energy scattering 344
15.6 Propagator for unstable particles 348
15.7 Global symmetries of the model 349

A Useful formulas **352**
A.1 Representation of γ-matrices 352
A.2 Traces of γ-matrices 354
A.3 The antisymmetric tensor 357
A.4 Useful integration formulas 358
A.4.1 Angular integrations in N-dimensional space 358
A.4.2 Momentum integration in loops 360

Index **363**