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GEOMETRIC DATA ANALYSIS

An Empirical Approach to Dimensionality Reduction
and the Study of Patterns



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