

# FUNDAMENTALS OF VIBRATIONS

LEONARD  
MEIROVITCH



McGRAW-HILL INTERNATIONAL EDITION  
Mechanical Engineering Series

# Contents

<b>Preface .....</b>	v
<b>Introduction .....</b>	xvi
<b>1 Concepts from Vibrations .....</b>	1
1.1 Newton's Laws .....	2
1.2 Moment of a Force and Angular Momentum .....	5
1.3 Work and Energy .....	6
1.4 Dynamics of Systems of Particles .....	10
1.5 Dynamics of Rigid Bodies .....	14
1.5.1 Pure translation relative to the inertial space .....	15
1.5.2 Pure rotation about a fixed point .....	16
1.5.3 General planar motion referred to the mass center .....	18
1.6 Kinetic Energy of Rigid Bodies in Planar Motion .....	21
1.6.1 Pure translation relative to the inertial space .....	21
1.6.2 Pure rotation about a fixed point .....	22
1.6.3 General planar motion referred to the mass center .....	22
1.7 Characteristics of Discrete System Components .....	23
1.8 Equivalent Springs, Dampers and Masses .....	27
1.9 Modeling of Mechanical Systems .....	39
1.10 System Differential Equations of Motion .....	44
1.11 Nature of Excitations .....	48
1.12 System and Response Characteristics. The Superposition Principle .....	53
1.13 Vibration about Equilibrium Points .....	57
1.14 Summary .....	66
Problems .....	67
<b>2 Response of Single-Degree-of-Freedom Systems to Initial Excitations .....</b>	80
2.1 Undamped Single-Degree-of-Freedom Systems. Harmonic Oscillator .....	81
2.2 Viscoelastically Damped Single-Degree-of-Freedom Systems .....	87
2.3 Measurement of Damping .....	94
2.4 Coulomb Damping. Dry Friction .....	98
2.5 Plotting the Response to Initial Excitations by MATLAB .....	101
2.6 Summary .....	102
Problems .....	103
<b>3 Response of Single-Degree-of-Freedom Systems to Harmonic and Periodic Excitations .....</b>	109
3.1 Response of Single-Degree-of-Freedom Systems to Harmonic Excitations .....	110

<b>3.2 Frequency Response Plots.....</b>	<b>114</b>
<b>3.3 Systems with Rotating Unbalanced Masses.....</b>	<b>120</b>
<b>3.4 Whirling of Rotating Shafts.....</b>	<b>122</b>
<b>3.5 Harmonic Motion of the Base.....</b>	<b>128</b>
<b>3.6 Vibration Isolation.....</b>	<b>131</b>
<b>3.7 Vibration Measuring Instruments.....</b>	<b>132</b>
<b>3.7.1 Accelerometers—high frequency instruments.....</b>	<b>133</b>
<b>3.7.2 Seismometers—low frequency instruments .....</b>	<b>135</b>
<b>3.8 Energy Dissipation. Structural Damping .....</b>	<b>137</b>
<b>3.9 Response to Periodic Excitations. Fourier Series.....</b>	<b>141</b>
<b>3.10 Frequency Response Plots by MATLAB.....</b>	<b>149</b>
<b>3.11 Summary.....</b>	<b>150</b>
<b>Problems .....</b>	<b>151</b>
<b>4 Response of Single-Degree-of-Freedom Systems to Nonperiodic Excitations .....</b>	<b>157</b>
<b>4.1 The Unit Impulse. Impulse Response.....</b>	<b>158</b>
<b>4.2 The Unit Step Function. Step Response.....</b>	<b>162</b>
<b>4.3 The Unit Ramp Function. Ramp Response.....</b>	<b>165</b>
<b>4.4 Response to Arbitrary Excitations. The Convolution Integral.....</b>	<b>168</b>
<b>4.5 Shock Spectrum .....</b>	<b>174</b>
<b>4.6 System Response by the Laplace Transformation Method. Transfer Function .....</b>	<b>177</b>
<b>4.7 General System Response .....</b>	<b>184</b>
<b>4.8 Response by the State Transition Matrix.....</b>	<b>186</b>
<b>4.9 Discrete-Time Systems. The Convolution Sum.....</b>	<b>189</b>
<b>4.10 Discrete-Time Response Using the Transition Matrix.....</b>	<b>198</b>
<b>4.11 Response by the Convolution Sum Using MATLAB.....</b>	<b>201</b>
<b>4.12 Response by the Discrete-Time Transition Matrix Using MATLAB .....</b>	<b>202</b>
<b>4.13 Summary .....</b>	<b>203</b>
<b>Problems .....</b>	<b>204</b>
<b>5 Two-Degree-of-Freedom Systems .....</b>	<b>208</b>
<b>5.1 System Configuration .....</b>	<b>209</b>
<b>5.2 The Equations of Motion of Two-Degree-of-Freedom Systems.....</b>	<b>211</b>
<b>5.3 Free Vibration of Undamped Systems. Natural Modes .....</b>	<b>215</b>
<b>5.4 Response to Initial Excitations .....</b>	<b>224</b>
<b>5.5 Coordinate Transformations. Coupling .....</b>	<b>225</b>
<b>5.6 Orthogonality of Modes. Natural Coordinates .....</b>	<b>229</b>
<b>5.7 Beat Phenomenon .....</b>	<b>233</b>
<b>5.8 Response of Two-Degree-of-Freedom Systems to Harmonic Excitations .....</b>	<b>238</b>
<b>5.9 Undamped Vibration Absorbers .....</b>	<b>240</b>
<b>5.10 Response of Two-Degree-of-Freedom Systems to Nonperiodic Excitations .....</b>	<b>243</b>
<b>5.11 Response to Nonperiodic Excitations by the Convolution Sum .....</b>	<b>248</b>
<b>5.12 Response to Initial Excitations by MATLAB .....</b>	<b>250</b>

5.13 Frequency Response Plots for Two-Degree-of-Freedom Systems by MATLAB .....	252
5.14 Response to a Rectangular Pulse by the Convolution Sum Using MATLAB .....	252
5.15 Summary .....	254
Problems .....	255
<b>6 Elements of Analytical Dynamics .....</b>	262
6.1 Degrees of Freedom and Generalized Coordinates .....	263
6.2 The Principle of Virtual Work .....	265
6.3 The Principle of D'Alembert .....	267
6.4 The Extended Hamilton's Principle .....	268
6.5 Lagrange's Equations .....	273
6.6 Summary .....	276
Problems .....	277
<b>7 Multi-Degree-of-Freedom Systems .....</b>	280
7.1 Equations of Motion for Linear Systems .....	281
7.2 Flexibility and Stiffness Influence Coefficients .....	285
7.3 Properties of the Stiffness and Mass Coefficients .....	290
7.4 Lagrange's Equations Linearized about Equilibrium .....	294
7.5 Linear Transformations. Coupling .....	297
7.6 Undamped Free Vibration. The Eigenvalue Problem .....	301
7.7 Orthogonality of Modal Vectors .....	309
7.8 Systems Admitting Rigid-Body Motions .....	310
7.9 Decomposition of the Response in Terms of Modal Vectors .....	316
7.10 Response to Initial Excitations by Modal Analysis .....	320
7.11 Eigenvalue Problem in Terms of a Single Symmetric Matrix .....	323
7.12 Geometric Interpretation of the Eigenvalue Problem .....	325
7.13 Rayleigh's Quotient and Its Properties .....	331
7.14 Response to Harmonic External Excitations .....	336
7.15 Response to External Excitations by Modal Analysis .....	337
7.15.1 Undamped systems .....	337
7.15.2 Systems with proportional damping .....	340
7.16 Systems with Arbitrary Viscous Damping .....	345
7.17 Discrete-Time Systems .....	355
7.18 Solution of the Eigenvalue Problem. MATLAB Programs .....	359
7.19 Response to Initial Excitations by Modal Analysis Using MATLAB .....	361
7.20 Response by the Discrete-Time Transition Matrix Using MATLAB .....	363
7.21 Summary .....	363
Problems .....	365
<b>8 Distributed-Parameter Systems: Exact Solutions .....</b>	374
8.1 Relation between Discrete and Distributed Systems. Transverse Vibration of Strings .....	375
8.2 Derivation of the String Vibration Problem by the Extended Hamilton Principle .....	380

<b>8.3</b>	<b>Bending Vibration of Beams .....</b>	<b>383</b>
<b>8.4</b>	<b>Free Vibration. The Differential Eigenvalue Problem.....</b>	<b>389</b>
<b>8.5</b>	<b>Orthogonality of Modes. Expansion Theorem.....</b>	<b>403</b>
<b>8.6</b>	<b>Systems with Lumped Masses at the Boundaries .....</b>	<b>408</b>
<b>8.7</b>	<b>Eigenvalue Problem and Expansion Theorem for Problems with Lumped Masses at the Boundaries .....</b>	<b>414</b>
<b>8.8</b>	<b>Rayleigh's Quotient. The Variational Approach to the Differential Eigenvalue Problem.....</b>	<b>423</b>
<b>8.9</b>	<b>Response to Initial Excitations .....</b>	<b>431</b>
<b>8.10</b>	<b>Response to External Excitations .....</b>	<b>439</b>
<b>8.11</b>	<b>Systems with External Forces at Boundaries .....</b>	<b>443</b>
<b>8.12</b>	<b>The Wave Equation.....</b>	<b>447</b>
<b>8.13</b>	<b>Traveling Waves in Rods of Finite Length.....</b>	<b>449</b>
<b>8.14</b>	<b>Summary .....</b>	<b>457</b>
	<b>Problems .....</b>	<b>458</b>
<b>9</b>	<b>Distributed-Parameter Systems: Approximate Methods .....</b>	<b>464</b>
<b>9.1</b>	<b>Discretization of Distributed-Parameter Systems by Lumping.....</b>	<b>465</b>
<b>9.2</b>	<b>Lumped-Parameter Method Using Influence Coefficients .....</b>	<b>468</b>
<b>9.3</b>	<b>Holzer's Method for Torsional Vibration .....</b>	<b>473</b>
<b>9.4</b>	<b>Myklestad's Method for Bending Vibration.....</b>	<b>484</b>
<b>9.5</b>	<b>Rayleigh's Principle .....</b>	<b>493</b>
<b>9.6</b>	<b>The Rayleigh-Ritz Method.....</b>	<b>499</b>
<b>9.7</b>	<b>An Enhanced Rayleigh-Ritz Method.....</b>	<b>516</b>
<b>9.8</b>	<b>The Assumed-Modes Method. System Response .....</b>	<b>523</b>
<b>9.9</b>	<b>The Galerkin Method .....</b>	<b>529</b>
<b>9.10</b>	<b>The Collocation Method.....</b>	<b>533</b>
<b>9.11</b>	<b>MATLAB Program for the Solution of the Eigenvalue Problem by the Rayleigh-Ritz Method .....</b>	<b>539</b>
<b>9.12</b>	<b>Summary .....</b>	<b>541</b>
	<b>Problems .....</b>	<b>543</b>
<b>10</b>	<b>The Finite Element Method .....</b>	<b>549</b>
<b>10.1</b>	<b>The Finite Element Method as a Rayleigh-Ritz Method.....</b>	<b>550</b>
<b>10.2</b>	<b>Strings, Rods and Shafts.....</b>	<b>554</b>
<b>10.3</b>	<b>Higher-Degree Interpolation Functions .....</b>	<b>563</b>
<b>10.4</b>	<b>Beams in Bending Vibration .....</b>	<b>574</b>
<b>10.5</b>	<b>Errors in the Eigenvalues .....</b>	<b>581</b>
<b>10.6</b>	<b>Finite Element Modeling of Trusses .....</b>	<b>583</b>
<b>10.7</b>	<b>Finite Element Modeling of Frames.....</b>	<b>597</b>
<b>10.8</b>	<b>System Response by the Finite Element Method .....</b>	<b>604</b>
<b>10.9</b>	<b>MATLAB Program for the Solution of the Eigenvalue Problem by the Finite Element Method .....</b>	<b>608</b>
<b>10.10</b>	<b>Summary .....</b>	<b>611</b>
	<b>Problems .....</b>	<b>612</b>

<b>11 Nonlinear Oscillations.....</b>	616
11.1 Fundamental Concepts in Stability. Equilibrium Points .....	617
11.2 Small Motions of Single-Degree-of-Freedom Systems from Equilibrium.....	628
11.3 Conservative Systems. Motions in the Large.....	639
11.4 Limit Cycles. The van der Pol Oscillator.....	644
11.5 The Fundamental Perturbation Technique.....	646
11.6 Secular Terms.....	649
11.7 Lindstedt's Method.....	652
11.8 Forced Oscillation of Quasi-Harmonic Systems. Jump Phenomenon.....	656
11.9 Subharmonics and Combination Harmonics .....	663
11.10 Systems with Time-Dependent Coefficients. Mathieu's Equation.....	667
11.11 Numerical Integration of the Equations of Motion. The Runge-Kutta Methods.....	672
11.12 Trajectories for the van der Pol Oscillator by MATLAB.....	679
11.13 Summary .....	680
Problems .....	682
<b>12 Random Vibrations.....</b>	685
12.1 Ensemble Averages. Stationary Random Processes .....	686
12.2 Time Averages. Ergodic Random Processes .....	689
12.3 Mean Square Values and Standard Deviation .....	691
12.4 Probability Density Functions .....	692
12.5 Description of Random Data in Terms of Probability Density Functions .....	699
12.6 Properties of Autocorrelation Functions .....	702
12.7 Response to Arbitrary Excitations by Fourier Transforms .....	703
12.8 Power Spectral Density Functions .....	708
12.9 Narrowband and Wideband Random Processes .....	710
12.10 Response of Linear Systems to Stationary Random Excitations .....	718
12.11 Response of Single-Degree-of-Freedom Systems to Random Excitations .....	722
12.12 Joint Probability Distribution of Two Random Variables .....	727
12.13 Joint Properties of Stationary Random Processes .....	730
12.14 Joint Properties of Ergodic Random Processes .....	733
12.15 Response Cross-Correlation Functions for Linear Systems .....	735
12.16 Response of Multi-Degree-of-Freedom Systems to Random Excitations .....	738
12.17 Response of Distributed-Parameter Systems to Random Excitations .....	743
12.18 Summary .....	746
Problems .....	748
<b>Appendix A. Fourier Series.....</b>	752
A.1 Orthogonal Sets of Functions .....	752
A.2 Trigonometric Series .....	754
A.3 Complex Form of Fourier Series .....	757
<b>Appendix B. Laplace Transformation .....</b>	759
B.1 Definition of the Laplace Transformation .....	759
B.2 Transformation of Derivatives .....	760

B.3 Transformation of Ordinary Differential Equations .....	760
B.4 The Inverse Laplace Transformation.....	761
B.5 Shifting Theorems .....	761
B.6 Method of Partial Fractions .....	762
B.7 The Convolution Integral. Borel's Theorem .....	765
B.8 Table of Laplace Transform Pairs .....	767
<b>Appendix C. Linear Algebra .....</b>	<b>768</b>
C.1 Matrices .....	768
C.1.1 Definitions .....	768
C.1.2 Matrix algebra.....	770
C.1.3 Determinant of a square matrix .....	772
C.1.4 Inverse of a matrix.....	774
C.1.5 Transpose, inverse and determinant of a product of matrices.....	775
C.1.6 Partitioned matrices .....	776
C.2 Vector Spaces.....	777
C.2.1 Definitions.....	777
C.2.2 Linear dependence .....	778
C.2.3 Bases and dimension of vector spaces .....	778
C.3 Linear Transformations.....	779
C.3.1 The concept of linear transformations .....	779
C.3.2 Solution of algebraic equations. Matrix inversion .....	781
<b>Bibliography .....</b>	<b>787</b>
<b>Index .....</b>	<b>789</b>