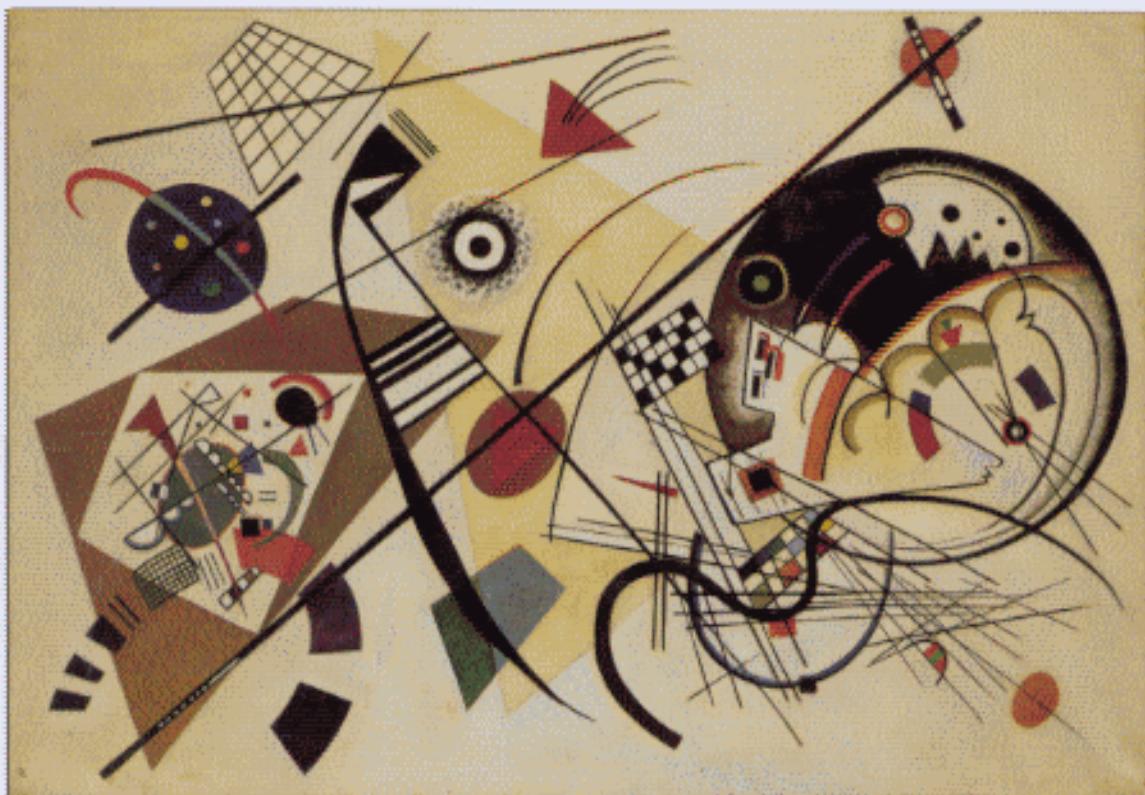


# Matrix Population Models

SECOND  
EDITION

CONSTRUCTION, ANALYSIS, AND INTERPRETATION



HAL CASWELL

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