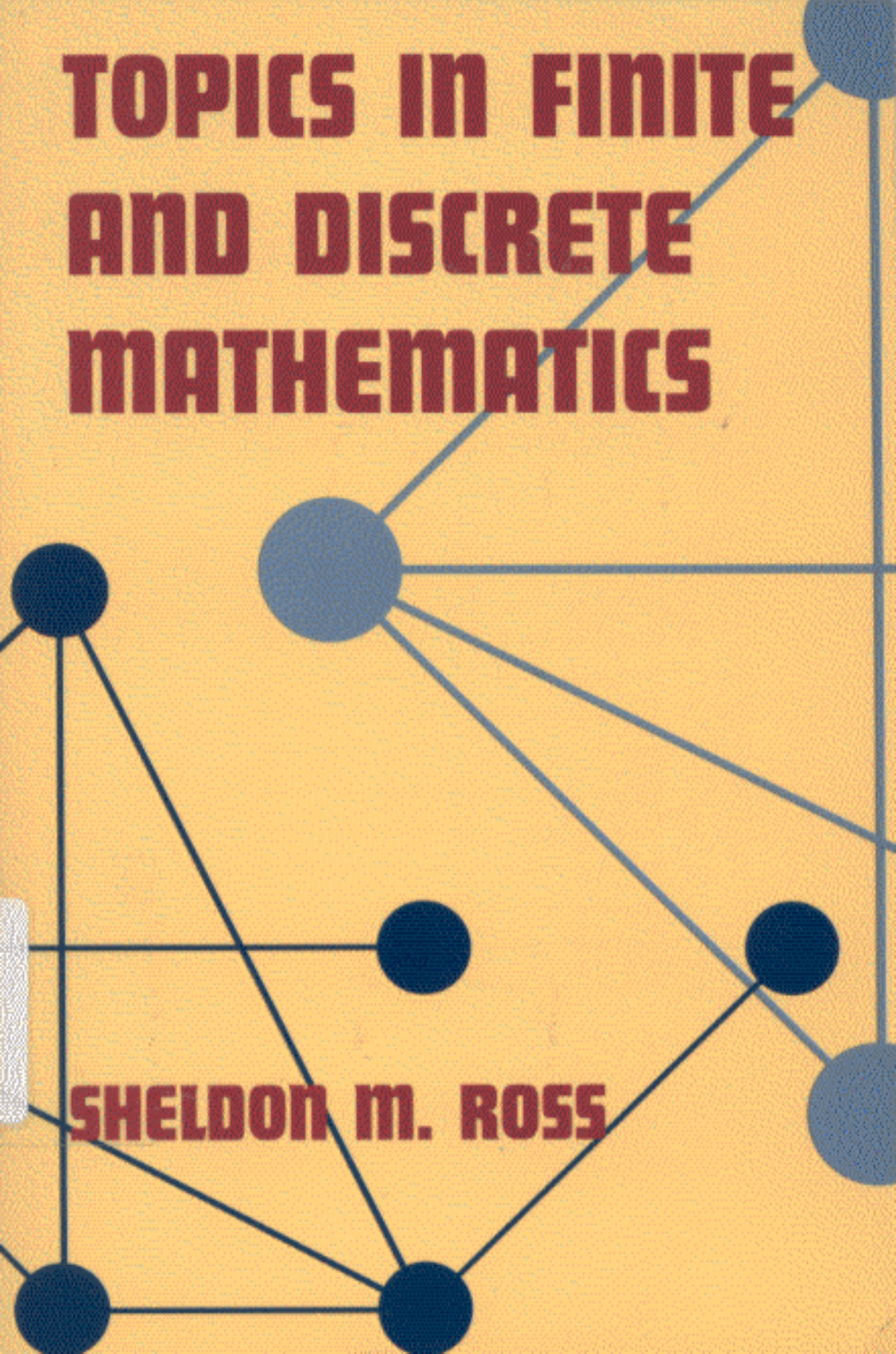
TOPICS IN FINITE AND DISCRETE MATHEMATICS



SHELDON M. ROSS

Contents

Preface *page* xi

1 Preliminaries 1
- 1.1 Sets 1
- 1.2 Summation 4
- 1.3 Mathematical Induction 8
- 1.4 Functions 17
- 1.5 The Division Algorithm 23
- 1.6 Exercises 28

2 Combinatorial Analysis 34
- 2.1 Introduction 34
- 2.2 The Basic Principle of Counting 34
- 2.3 Permutations 36
- 2.4 Combinations 40
- 2.5 Counting the Number of Solutions 45
- 2.6 The Inclusion–Exclusion Identity 47
- 2.7 Using Recursion Equations 52
- 2.8 The Pigeonhole Principle 61
- 2.9 Exercises 63

3 Probability 70
- 3.1 Probabilities and Events 70
- 3.2 Probability Experiments Having Equally Likely Outcomes 74
- 3.3 Conditional Probability 77
- 3.4 Computing Probabilities by Conditioning 80
- 3.5 Random Variables and Expected Values 85
- 3.6 Exercises 94

4 Mathematics of Finance 97
- 4.1 Interest Rates 97
- 4.2 Present Value Analysis 100
- 4.3 Pricing Contracts via Arbitrage 104
  - 4.3.1 An Example in Options Pricing 104
  - 4.3.2 Other Examples of Pricing via Arbitrage 107

4.4 The Arbitrage Theorem 111
4.5 The Multiperiod Binomial Model 116
4.5.1 The Black–Scholes Option Pricing Formula 120
4.6 Exercises 121

5 Graphs and Trees 124
5.1 Graphs 124
5.2 Trees 127
5.3 The Minimum Spanning Tree Problem 131
5.4 Cliques and Independent Sets 134
5.5 Euler Graphs 142
5.6 Exercises 144

6 Directed Graphs 150
6.1 Directed Graphs 150
6.2 The Maximum Flow Problem 150
6.3 Applications of the Maximum Flow Problem 160
6.3.1 The Assignment Problem 160
6.3.2 The Tournament Win Problem 163
6.3.3 The Transshipment Problem 166
6.3.4 An Equipment Selection Problem 167
6.4 Shortest Path in Digraphs 170
6.5 Exercises 175

7 Linear Programming 180
7.1 The Standard Linear Programming Problem 180
7.2 Transforming to the Standard Form 184
7.2.1 Minimization and Wrong-Way Inequality Constraints 185
7.2.2 Problems with Variables Unconstrained in Sign 186
7.3 The Dual Linear Programming Problem 188
7.4 Game Theory 194
7.5 Exercises 199

8 Sorting and Searching 203
8.1 Introduction to Sorting 203
8.2 The Bubble Sort 203
8.3 The Quicksort Algorithm 206
8.4 Merge Sorts 209
8.5 Sequential Searching 210
8.6 Binary Searches and Rooted Trees 212
8.7 Exercises 218

9 Statistics 220
9.1 Introduction 220
9.2 Frequency Tables and Graphs 220
9.3 Summarizing Data Sets 223
9.3.1 Sample Mean, Sample Median, and Sample Mode 223
9.3.2 Sample Variance and Sample Standard Deviation 225
9.4 Chebyshev's Inequality 227
9.5 Paired Data Sets and the Sample Correlation Coefficient 229
9.6 Testing Statistical Hypotheses 232
9.7 Exercises 233

10 Groups and Permutations 237
10.1 Permutations and Groups 237
10.2 Permutation Graphs 243
10.3 Subgroups 244
10.4 Lagrange's Theorem 249
10.5 The Alternating Subgroup 254
10.6 Exercises 259

Index 263