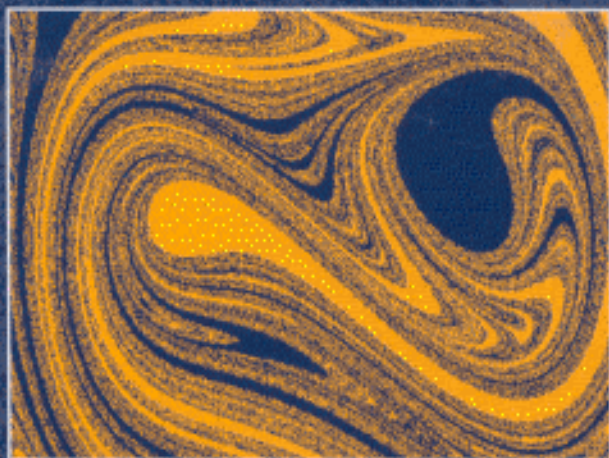


Chaos and Nonlinear Dynamics



**An Introduction for
Scientists and Engineers**

Second Edition

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Contents

First Edition Preface	v
First Edition Acknowledgments	xi
Second Edition Preface	xiii
Second Edition Acknowledgments	xv
I. THE PHENOMENOLOGY OF CHAOS	1
1 Three Chaotic Systems	3
1.1 Prelude	3
1.2 Linear and Nonlinear Systems	4
1.3 A Nonlinear Electrical System	8
1.4 A Mathematical Model of Biological Population Growth	17
1.5 A Model of Convecting Fluids: The Lorenz Model	27
1.6 Determinism, Unpredictability, and Divergence of Trajectories	37
1.7 Summary and Conclusions	39
1.8 Further Reading	40
2 The Universality of Chaos	47
2.1 Introduction	47
2.2 The Feigenbaum Numbers	47
2.3 Convergence Ratio for Real Systems	51
2.4 Using δ to Make Predictions	53
2.5 Feigenbaum Size Scaling	55
2.6 Self-Similarity	56
2.7 Other Universal Features	57
2.8 Models and the Universality of Chaos	58
2.9 Computers and Chaos	61
2.10 Further Reading	63
2.11 Computer Exercises	64
II. TOWARD A THEORY OF NONLINEAR DYNAMICS AND CHAOS	69
3 Dynamics in State Space: One and Two Dimensions	71
3.1 Introduction	71

3.2	State Space	72
3.3	Systems Described by First-Order Differential Equations	74
3.4	The No-Intersection Theorem	77
3.5	Dissipative Systems and Attractors	78
3.6	One-Dimensional State Space	79
3.7	Taylor Series Linearization Near Fixed Points	83
3.8	Trajectories in a One-Dimensional State Space	84
3.9	Dissipation Revisited	86
3.10	Two-Dimensional State Space	87
3.11	Two-Dimensional State Space: The General Case	91
3.12	Dynamics and Complex Characteristic Values	94
3.13	Dissipation and the Divergence Theorem	96
3.14	The Jacobian Matrix for Characteristic Values	97
3.15	Limit Cycles	100
3.16	Poincaré Sections and the Stability of Limit Cycles	102
3.17	Bifurcation Theory	106
3.18	Summary	113
3.19	Further Reading	114
3.20	Computer Exercises	116
4	Three-Dimensional State Space and Chaos	117
4.1	Overview	117
4.2	Heuristics	118
4.3	Routes to Chaos	121
4.4	Three-Dimensional Dynamical Systems	123
4.5	Fixed Points in Three Dimensions	124
4.6	Limit Cycles and Poincaré Sections	128
4.7	Quasi-Periodic Behavior	134
4.8	The Routes to Chaos I: Period-Doubling	136
4.9	The Routes to Chaos II: Quasi-Periodicity	137
4.10	The Routes to Chaos III: Intermittency and Crises	138
4.11	The Routes to Chaos IV: Chaotic Transients and Homoclinic Orbits	138
4.12	Homoclinic Tangles and Horseshoes	146
4.13	Lyapunov Exponents and Chaos	148
4.14	Further Reading	154
4.15	Computer Exercises	155
5	Iterated Maps	157
5.1	Introduction	157
5.2	Poincaré Sections and Iterated Maps	158
5.3	One-Dimensional Iterated Maps	163
5.4	Bifurcations in Iterated Maps: Period-Doubling, Chaos, and Lyapunov Exponents	166

5.5	Qualitative Universal Behavior: The U -Sequence	173
5.6	Feigenbaum Universality	183
5.7	Tent Map	185
5.8	Shift Maps and Symbolic Dynamics	188
5.9	The Gaussian Map	192
5.10	Two-Dimensional Iterated Maps	197
5.11	The Smale Horseshoe Map	199
5.12	Summary	204
5.13	Further Reading	204
5.14	Computer Exercises	207
6	Quasi-Periodicity and Chaos	210
6.1	Introduction	210
6.2	Quasi-Periodicity and Poincaré Sections	212
6.3	Quasi-Periodic Route to Chaos	214
6.4	Universality in the Quasi-Periodic Route to Chaos	215
6.5	Frequency-Locking	217
6.6	Winding Numbers	218
6.7	Circle Map	219
6.8	The Devil's Staircase and the Farey Tree	227
6.9	Continued Fractions and Fibonacci Numbers	231
6.10	On to Chaos and Universality	234
6.11	Some Applications	240
6.12	Further Reading	246
6.13	Computer Exercises	249
7	Intermittency and Crises	250
7.1	Introduction	250
7.2	What Is Intermittency?	250
7.3	The Cause of Intermittency	252
7.4	Quantitative Theory of Intermittency	256
7.5	Types of Intermittency and Experimental Observations	259
7.6	Crises	260
7.7	Some Conclusions	267
7.8	Further Reading	268
7.9	Computer Exercises	270
8	Hamiltonian Systems	272
8.1	Introduction	272
8.2	Hamilton's Equations and the Hamiltonian	273
8.3	Phase Space	276
8.4	Constants of the Motion and Integrable Hamiltonians	279
8.5	Nonintegrable Systems, the KAM Theorem, and Period-Doubling	289
8.6	The Hénon–Heiles Hamiltonian	296

8.7	The Chirikov Standard Map	303
8.8	The Arnold Cat Map	308
8.9	The Dissipative Standard Map	309
8.10	Applications of Hamiltonian Dynamics	311
8.11	Further Reading	313
8.12	Computer Exercises	316
III. MEASURES OF CHAOS		317
9	Quantifying Chaos	319
9.1	Introduction	319
9.2	Time-Series of Dynamical Variables	320
9.3	Lyapunov Exponents	323
9.4	Universal Scaling of the Lyapunov Exponent	327
9.5	Invariant Measure	330
9.6	Kolmogorov-Sinai Entropy	335
9.7	Fractal Dimension(s)	341
9.8	Correlation Dimension and a Computational Case History	354
9.9	Comments and Conclusions	368
9.10	Further Reading	369
9.11	Computer Exercises	374
10	Many Dimensions and Multifractals	375
10.1	General Comments and Introduction	375
10.2	Embedding (Reconstruction) Spaces	376
10.3	Practical Considerations for Embedding Calculations	383
10.4	Generalized Dimensions and Generalized Correlation Sums	389
10.5	Multifractals and the Spectrum of Scaling Indices $f(\alpha)$	393
10.6	Generalized Entropy and the $g(\Lambda)$ Spectrum	404
10.7	Characterizing Chaos via Periodic Orbits	413
10.8	*Statistical Mechanical and Thermodynamic Formalism	415
10.9	Wavelet Analysis, q -Calculus, and Related Topics	420
10.10	Summary	421
10.11	Further Reading	422
10.12	Computer Exercises	429
IV. SPECIAL TOPICS		431
11	Pattern Formation and Spatiotemporal Chaos	433
11.1	Introduction	433
11.2	Two-Dimensional Fluid Flow	436
11.3	Coupled-Oscillator Models, Cellular Automata, and Networks	442

Contents	xxi
11.4 Transport Models	450
11.5 Reaction-Diffusion Systems: A Paradigm for Pattern Formation	460
11.6 Diffusion-Limited Aggregation, Dielectric Breakdown, and Viscous Fingering: Fractals Revisited	471
11.7 Self-Organized Criticality: The Physics of Fractals?	477
11.8 Summary	479
11.9 Further Reading	480
11.10 Computer Exercises	489
12 Quantum Chaos, The Theory of Complexity, and Other Topics	490
12.1 Introduction	490
12.2 Quantum Mechanics and Chaos	490
12.3 Chaos and Algorithmic Complexity	508
12.4 Miscellaneous Topics: Piece-wise Linear Models, Time-Delay Models, Information Theory, Stochastic Resonance, Computer Networks, Controlling and Synchronizing Chaos	510
12.5 Roll Your Own: Some Simple Chaos Experiments	517
12.6 General Comments and Overview: The Future of Chaos	517
12.7 Further Reading	519
Appendix A: Fourier Power Spectra	533
Appendix B: Bifurcation Theory	541
Appendix C: The Lorenz Model	547
Appendix D: The Research Literature on Chaos	559
Appendix E: Computer Programs	560
Appendix F: Theory of the Universal Feigenbaum Numbers	568
Appendix G: The Duffing Double-Well Oscillator	579
Appendix H: Other Universal Features for One-Dimensional Iterated Maps	584
Appendix I: The van der Pol Oscillator	589
Appendix J: Simple Laser Dynamics Models	598
References	605
Index	643