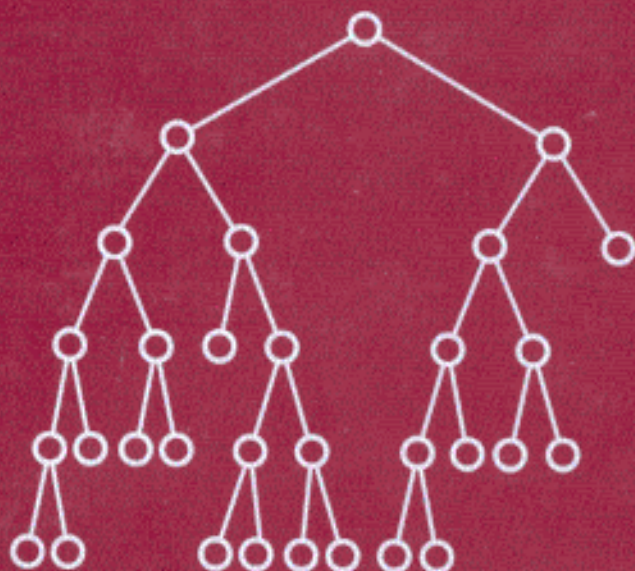


LOGIC-BASED METHODS FOR OPTIMIZATION

COMBINING OPTIMIZATION AND
CONSTRAINT SATISFACTION



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