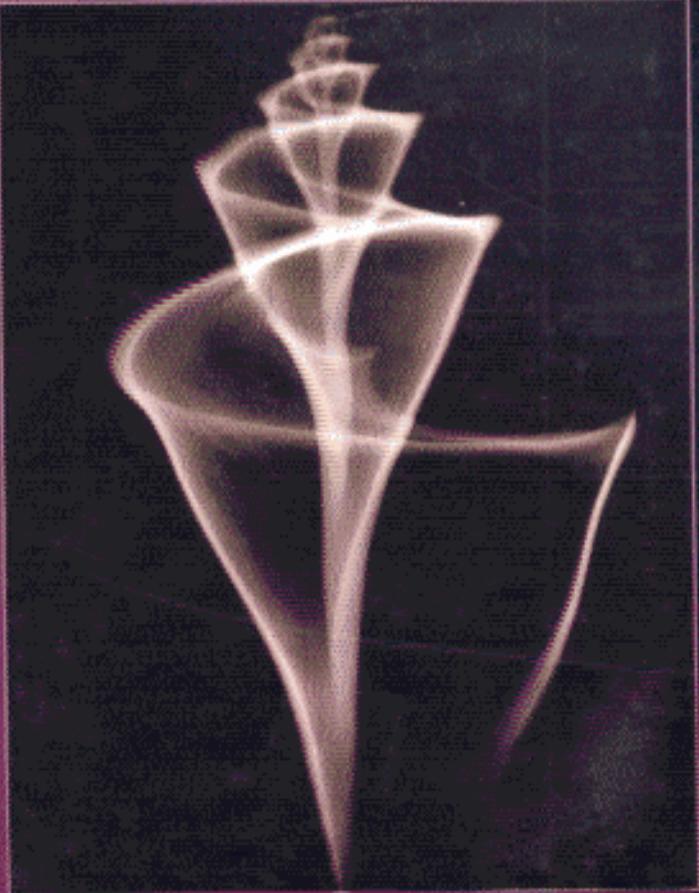


A nalysis

of Algorithms

An Active Learning Approach



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JONES AND BARTLETT COMPUTER SCIENCE

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