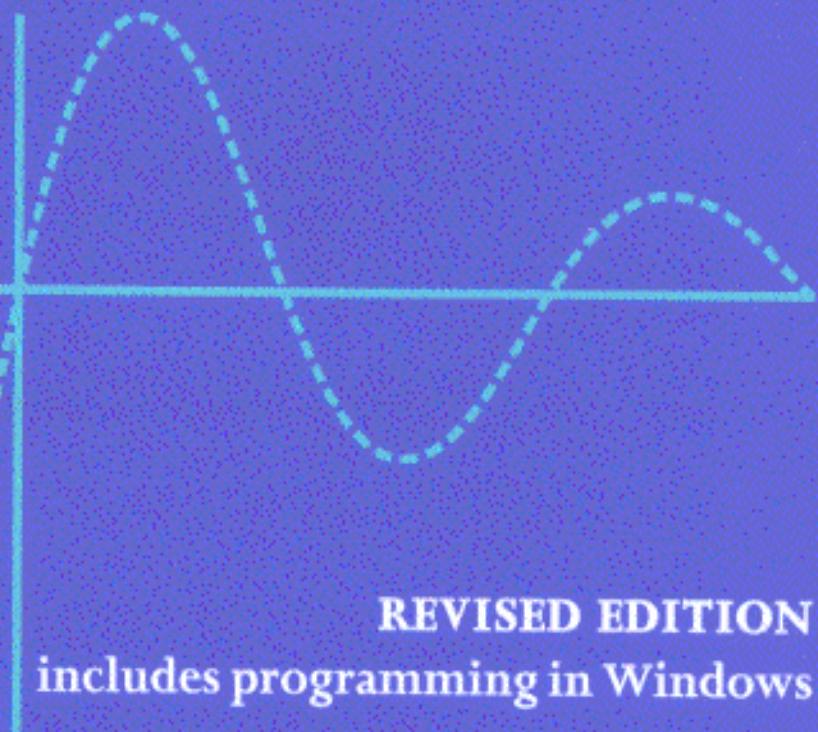


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An Introduction to Numerical Methods in C++



REVISED EDITION
includes programming in Windows

B. H. Flowers

Contents

1 Preliminaries	1
1.1 Elementary input and output	1
1.1.1 Strings	1
1.1.2 Numbers	3
1.1.3 Standard error output	4
1.2 Comments	5
1.3 Basic data types	6
1.3.1 Integer types	6
1.3.2 Floating point types	7
1.3.3 Void type	8
1.4 Derived types	8
1.4.1 Constants	9
1.4.2 Enumerations	9
1.4.3 Structures	10
1.5 Pointers	11
1.5.1 Pointer to void	13
1.5.2 Constants and pointers	13
1.5.3 Pointers to structures	14
1.5.4 Arrays	14
1.6 References	16
1.7 Preprocessor directives	17
1.7.1 Token replacement	17
1.7.2 Conditional directives	18
1.7.3 File inclusion	18
1.8 Keywords and identifiers	19

2 Expressions, statements and functions	20
2.1 Expressions	21
2.1.1 Arithmetic expressions	21
2.1.2 Relational expressions	22
2.1.3 Logical expressions	22
2.1.4 Bitwise expressions	22
2.1.5 Comma expressions	23
2.1.6 Other expressions	23
2.1.7 Operator precedence	23
2.2 Statements	25
2.2.1 Declaration and initialization	25
2.2.2 Assignments	26
2.2.3 Compound assignments	26
2.2.4 Increments and decrements	26
2.3 Compound statements	28
2.3.1 Scope	29
2.3.2 Lifetime	30
2.4 Conditional statements	30
2.4.1 if-else statements	30
2.4.2 while statements	33
2.4.3 do statements	33
2.4.4 for statements	34
2.4.5 break and continue statements	34
2.4.6 goto statements	35
2.4.7 switch statements	36
2.5 Functions	37
2.5.1 Inline functions	41
2.5.2 Passing arguments to functions	42
2.5.3 Functions returning void	42
2.5.4 Reference arguments	43
2.5.5 Pointer arguments	44
2.5.6 Overloading function names	44
2.5.7 Function templates	45
2.6 Recursive functions	46
2.7 Static local variables	49
2.8 Pointers and functions	50
2.9 Function types	51
2.10 The function main	52

3 Errors, theorems and speed	55
3.1 Truncation	55
3.2 Rounding errors	55
3.3 Quadratic equations	57
3.4 Floating point equality	58
3.5 Conditioning and stability	59
3.6 Local and global errors	60
3.7 Basic theorems	60
3.8 Rates of convergence	62
3.9 Reciprocals without division	63
3.10 Speed of computation	64
3.11 Speed and recursion	65
3.12 Mistakes	68
4 Roots of non-linear equations	69
4.1 Bisection method	69
4.1.1 Simple recursive procedure	69
4.1.2 Refined recursive procedure	70
4.1.3 Non-recursive procedure	73
4.2 Regula falsi method	74
4.3 Secant method	76
4.4 Convergence	77
4.5 Fixed point method	79
4.6 Choice of fixed point function	81
4.7 Newton's method	82
4.8 Multiple roots	83
4.9 Aitken's extrapolation	84
4.10 Roots of polynomials	85
4.10.1 Horner's algorithm	86
4.10.2 Stability	87
4.11 Maxima and minima of functions	88
4.12 Some standard things	91

5 Classes

5.1	Complex numbers	94
5.1.1	Declaration	95
5.1.2	Implementation	96
5.1.3	Friend functions	99
5.1.4	Operator functions	100
5.1.5	Complex roots of quadratic equations	103
5.2	Class string	104
5.2.1	Declaration	104
5.2.2	Implementation	105
5.2.3	Assignment	107
5.2.4	Indexing	108
5.2.5	Concatenation	110
5.2.6	Input and output	112
5.2.7	Example	112
5.3	Static members	113
5.4	Class arrays	115
5.4.1	Array of complex numbers	116
5.4.2	Array of strings	117
5.4.3	Array template	118
5.5	Classes and header files	119

6 Derived classes and streams

6.1	The base class tuple	120
6.1.1	The derived class point	121
6.1.2	The derived class complex	124
6.2	Derivation and access	125
6.3	Class string revisited	127
6.4	Streams	131
6.4.1	The stream buffer	131
6.4.2	Output streams	132
6.4.3	Input streams	133
6.4.4	Formatted input and output	133
6.4.5	Tabulation	137
6.5	File input and output	139
6.5.1	Text files	141
6.5.2	Binary files	143

7 Integer arithmetic

7.1	Prime numbers	145
7.1.1	A table of primes	146
7.1.2	Prime factors	148
7.1.3	The greatest common divisor	150
7.2	Rational numbers	151
7.2.1	The class rational	152
7.2.2	The harmonic numbers	157
7.2.3	Bernoulli's numbers	158
7.3	Congruences and residues	160
7.3.1	Fermat's theorem	161
7.3.2	Periodic sequences	162
7.4	Random numbers	162
7.4.1	Linear congruence generators	163
7.4.2	Choice of parameters	164
7.4.3	Sequential potency	166
7.4.4	Practical generators	168
7.4.5	Shuffling	169
7.4.6	Example—Monte Carlo integration	170

8 Tests of randomness

8.1	Even distribution test	173
8.2	Serial correlation test	176
8.3	The spectral test	177
8.3.1	Minimization process	179
8.3.2	The case $k = 2$	182
8.3.3	The case $k = 3$	183
8.3.4	Results	187

9 Vectors and matrices

9.1	The class vector	188
9.2	The class matrix	195
9.3	Operations of linear algebra	200

10 Direct solution of linear equations	204
10.1 Gaussian elimination	205
10.1.1 Algorithms	205
10.1.2 Speed of computation	207
10.2 Refinement	208
10.2.1 Pivoting	208
10.2.2 The case of the vanishing determinant	210
10.2.3 Minimizing the arithmetic	210
10.2.4 Refined gauss	211
10.2.5 Improved speed	212
10.2.6 Scaling	212
10.3 Matrix decomposition	213
10.4 Crout algorithm	215
10.4.1 Simple Crout	215
10.4.2 Speed	217
10.4.3 Crout with pivoting	218
10.5 The inverse matrix	220
10.6 Tridiagonal equations	222
10.7 Modular programming	225
11 Errors in matrix manipulation	227
11.1 Norms	227
11.2 Convergence	230
11.3 Error estimation	231
11.3.1 Random perturbations	233
11.4 Iterative refinement	234
12 Iterative solutions of systems of equations	236
12.1 Gauss–Jacobi iteration	236
12.2 Gauss–Seidel iteration	239
12.3 Jacobi and Seidel compared	241
12.4 Successive over-relaxation	242
12.5 Speed of computation	243
12.6 Roots of systems of non-linear equations	246

13 Matrix eigenvalue problems	250
13.1 General theory	251
13.1.1 Locating eigenvalues	252
13.1.2 Eigenvalue stability and errors of computation	254
13.2 The power method	255
13.3 Inverse power method	261
13.4 Jacobi rotation method	263
13.5 Householder's method	268
13.5.1 Reduction to tridiagonal form	269
13.5.2 Francis diagonalization	273
14 Interpolation and data fitting	280
14.1 Lagrangian interpolation	281
14.1.1 Lagrangian algorithm	282
14.1.2 Errors of interpolation	283
14.1.3 Neville's algorithm	286
14.1.4 Divided differences	288
14.1.5 Inverse interpolation	289
14.2 Cubic spline piecewise interpolation	289
14.2.1 Cubic spline routine	290
14.2.2 Lagrange polynomial and cubic spline compared	292
14.3 Data fitting	293
14.3.1 Least squares approximation	294
14.3.2 Other approximations	296
14.4 Sorting	297
14.4.1 Quicksort	299
14.4.2 Heapsort	302
15 Graphics	305
15.1 Text mode	305
15.2 Graphics mode	310
15.2.1 Classes point and pixel	311
15.2.2 Block size	314
15.2.3 Opening and closing graphics mode	316
15.2.4 Presentation	316
15.2.5 Drawing a function	318
15.2.6 Scanning the screen	320

15.3	The complex plane: quadratic Julia sets	321
15.4	Dynamic use of colour: the Mandelbrot set	325
15.5	Bezier curves	328

16 Differentiation and integration

16.1	Differentiation	334
16.2	Richardson extrapolation	338
16.3	Integration	339
16.4	Composite formulae	343
16.5	Asymptotic errors	346
16.6	Adaptive integration	347
16.7	Romberg integration	349
16.8	Improper integrals	354
16.9	Multiple integrals	355

17 Orthogonal polynomials

17.1	The Gram–Schmidt process	357
17.2	Zeros of orthogonal polynomials	359
17.3	Particular systems	360
17.3.1	Legendre polynomials	360
17.3.2	Chebyshev polynomials	362
17.3.3	Laguerre polynomials	363
17.3.4	Hermite polynomials	364
17.4	Approximation of functions	364
17.4.1	Taylor approximation	366
17.4.2	Legendre approximation	366
17.4.3	Chebyshev approximation	368
17.5	Minimax approximation	369
17.6	Gaussian quadrature	372
17.6.1	Computation of nodes	374
17.6.2	Computation of weights	375
17.6.3	Nodes and weights combined	376
17.6.4	An example	380

18 Differential equations

18.1	Existence and uniqueness	383
18.2	Stability and conditioning	385

18.3	Euler's method	387
18.4	Higher order methods	391
18.5	Runge–Kutta methods	392
18.5.1	Convergence	395
18.5.2	Adaptation	396
18.6	Multiple-step methods	399
18.6.1	Two-step midpoint method	399
18.6.2	Other two-step methods	404
18.7	Predictor–corrector methods	409
18.8	Domain of existence	411
19	More about differential equations	413
19.1	Systems of ordinary differential equations	413
19.1.1	Runge–Kutta method	414
19.1.2	Lotka–Volterra equations	415
19.2	Second-order initial value problems	418
19.3	Second-order boundary value problems	420
19.3.1	The shooting method	420
19.3.2	The linear finite difference method	423
19.3.3	Extrapolated linear finite difference method	426
19.3.4	The non-linear finite difference method	426
19.4	Variational methods	428
19.5	Eigenvalue problems	431
19.6	Partial differential equations	433
20	Recursive data types—lists	438
20.1	The base class <code>list</code>	441
20.2	List of class template	448
20.3	The travelling salesman	451
20.4	Multiple byte integers	461
21	Elements of Fourier analysis	471
21.1	Fourier series	471
21.2	Periodic functions	473
21.3	Discrete Fourier transforms	475
21.4	Fast Fourier transforms	476

Addendum Programming in Windows	481
A1 The Window classes TApplication and TWindow	483
A2 The class TWindow and the function Paint	487
A3 Programming mouse buttons	491
A4 An input dialogue box	492
A5 A multiple document interface	494
A6 The class TScreen	501
A7 Drawing a function	511
A8 Mapping the screen	515
A9 Ordinary differential equations	516
A10 Two simultaneous differential equations	523
Bibliography	533
Appendix A standard.h	535
Appendix B restring.h	537
Appendix C rational.h	538
Appendix D random.h	539
Appendix E vecmat.h	540
Appendix F stdgraph.h	542
Appendix G list.h	544
Appendix H screen.h	546
Index	549