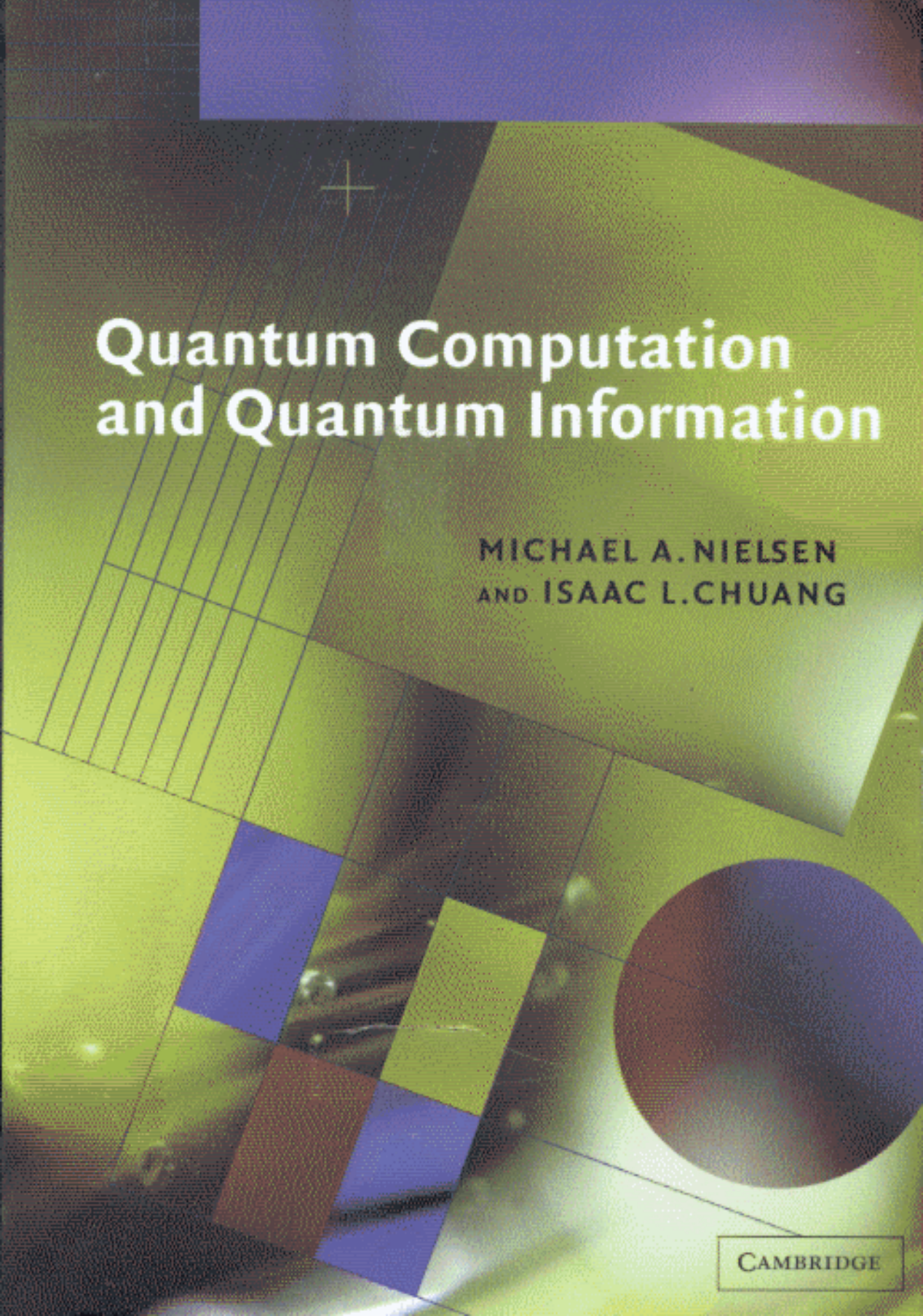




Quantum Computation and Quantum Information

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The background of the book cover is a complex, abstract geometric design. It features a grid of squares and rectangles in various shades of green, yellow, and blue. A large, dark blue circle is positioned in the lower right quadrant. The overall effect is a modern, mathematical aesthetic.

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