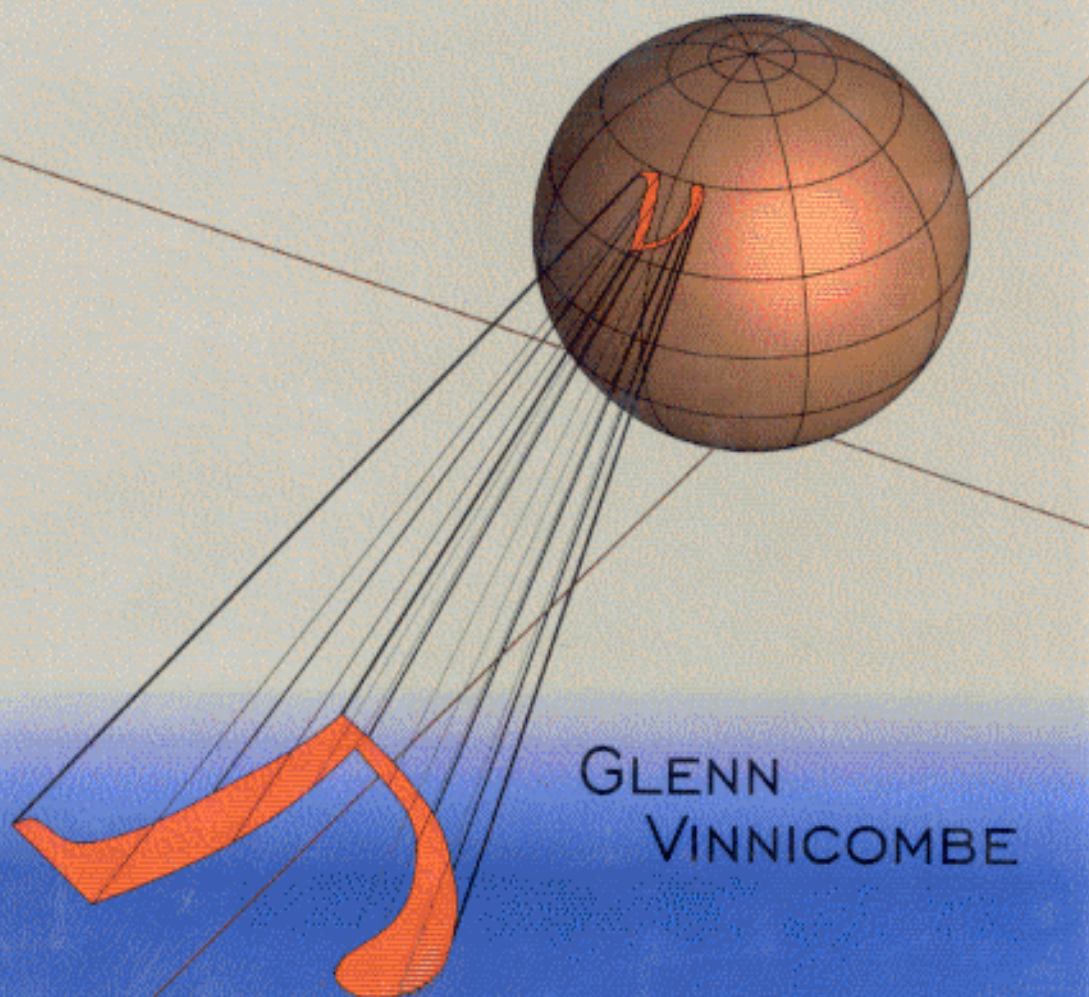


UNCERTAINTY AND FEEDBACK

$\mathcal{H}_\infty$ loop-shaping and the v-gap metric



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