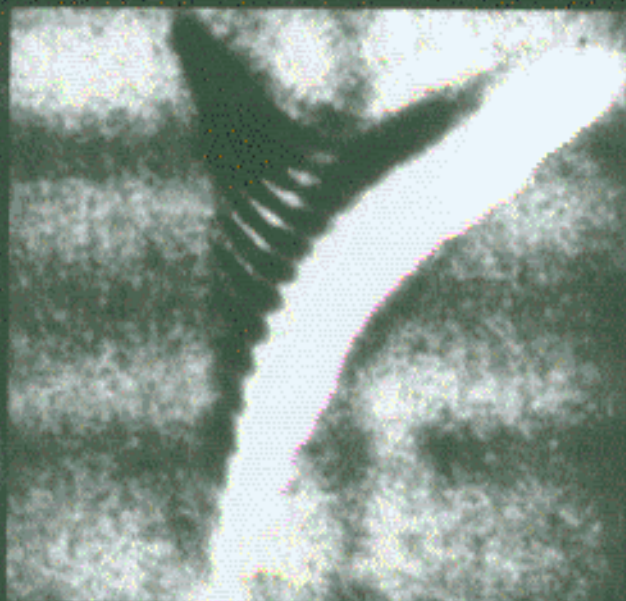


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Dynamical Theory of X-Ray Diffraction

André Authier



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Contents

I	Background and basic results	1
1	Historical developments	3
1.1	Prologue	3
1.2	The discovery of X-ray diffraction	4
1.3	The geometrical theory of diffraction	5
1.4	Darwin's dynamical theory of diffraction	6
1.5	Extinction theories	8
1.6	Ewald's dynamical theory	11
1.7	Early confirmations of the dynamical theory	13
1.8	Laue's dynamical theory	14
1.9	<i>Umweganregung</i> and <i>Aufhellung</i>	14
1.10	The properties of wavefields	16
1.10.1	Anomalous absorption (the Borrmann effect)	16
1.10.2	Wavefield trajectories	20
1.10.3	Pendellösung	23
1.11	Diffraction by deformed crystals	25
1.12	Modern times	26
2	Properties of the electromagnetic field—propagation and scattering	28
2.1	Maxwell's equations	28
2.2	The electrodynamic potentials in vacuum	29
2.2.1	The vector and scalar potentials	29
2.2.2	The retarded potentials	30
2.3	The electrodynamic potentials in polarized media	31
2.4	Hertz vectors (polarization potentials)	31
2.5	Propagation of an electromagnetic wave in vacuum	33
2.6	Scattering of X-rays by an electron	33
2.7	Polarizability of matter for X-rays	36
2.7.1	Elementary dispersion theory	36
2.7.2	Fourier expansion of the polarizability	37
2.7.3	Index of refraction	41
2.7.4	Absorption	41
2.8	Ewald's dispersion theory	43
2.9	Propagation equation of an electromagnetic wave in materials in Laue's dynamical theory	49
2.9.1	Laue's basic assumption	49

2.9.2	Propagation equation	49
2.10	Specular reflection—Fresnel relations	50
3	Geometrical theory of X-ray diffraction	57
3.1	Classical scattering by an electron—polarization	57
3.2	Amplitude diffracted by a periodic electron distribution	58
3.3	Intensity diffracted by a small crystal	61
3.4	Reflectivity	63
3.5	Integrated intensity	65
3.6	Mosaic crystals	67
4	Elementary dynamical theory	68
4.1	Limitations of the geometrical theory	68
4.2	Introduction of the dispersion surface	69
4.3	Analogy with the band theory of solids	71
4.4	Propagation equation	73
4.5	Fundamental equations of dynamical theory	74
4.6	Amplitude ratio of the refracted and reflected waves	79
4.7	Solutions of plane-wave dynamical theory	80
4.7.1	Boundary conditions	80
4.7.2	Departure from Bragg's angle of the incident wave	81
4.7.3	Transmission and reflection geometries	82
4.7.4	Deviation parameter	85
4.7.5	Determination of the tiepoints	85
4.7.6	Effective absorption coefficient	87
4.8	The diffracted waves in the transmission geometry	88
4.8.1	Double refraction	88
4.8.2	Boundary conditions for the amplitudes at the entrance surface	88
4.8.3	Intensities of the reflected and refracted waves	89
4.8.4	Anomalous absorption	90
4.8.5	Boundary conditions at the exit surface	92
4.8.6	Reflectivity	94
4.8.7	Pendellösung	96
4.8.8	Integrated intensity	98
4.9	The diffracted waves in the reflection geometry	99
4.9.1	Tiepoints	99
4.9.2	Thick crystals—total reflection	99
4.9.3	Thin crystals	102
4.10	Influence of the asymmetry on the position and width of the rocking curve and of the angular distribution of the reflected beam	104
4.11	Comparison with geometrical theory	107
4.12	Dynamical diffraction by quasicrystals	110

II	Advanced dynamical theory	113
5	Properties of wavefields	115
5.1	Relations between the field vectors	115
5.2	Fundamental equations of the dynamical theory	117
5.3	The dispersion equation in the two-beam case	118
5.4	Poynting vector of the wavefields	121
5.5	Determination of the tiepoints—geometrical interpretation of the deviation parameter	123
5.5.1	Boundary condition for the wavevectors	123
5.5.2	Deviation from Bragg's angle of the middle of the reflection domain	125
5.5.3	Coordinates of the tiepoint	126
5.5.4	Deviation parameter, Pendellösung distance and Darwin width in the transmission geometry	128
5.5.5	Deviation parameter, extinction distance, penetration depth and Darwin width in the reflection geometry	132
5.5.6	Index of refraction for dynamical diffraction	135
5.6	The deviation parameter in absorbing crystals	136
5.7	Amplitude ratio of the refracted and reflected waves	136
5.7.1	Phase of the amplitude ratio in the transmission geometry	137
5.7.2	Phase of the amplitude ratio in the reflection geometry	138
5.8	Anomalous absorption	139
5.8.1	Effective absorption coefficient in the transmission geometry	139
5.8.2	Absorption coefficient in the propagation direction	141
5.8.3	Discussion of anomalous absorption—properties of the standing wavefield	142
5.8.4	Anomalous absorption in the reflection geometry—penetration depth	147
5.9	Dispersion surface when the Bragg angle is close to $\pi/2$	148
5.9.1	Deviation from Bragg's angle and Darwin width	148
5.9.2	Dispersion surface	151
5.9.3	Penetration depth	153
5.9.4	Applications	154
6	Intensities of plane waves in the transmission geometry	155
6.1	Boundary conditions for the amplitudes at the entrance surface	155
6.2	Amplitudes of the refracted and reflected waves	157

6.3	Boundary conditions for the wavevectors at the exit surface	161
6.3.1	Condition for the existence of two outgoing waves	161
6.3.2	Wavevectors of the outgoing waves (Laue–Laue geometry)	163
6.3.3	Laue–Bragg geometry	165
6.4	Rocking curves of the reflected and refracted beams	166
6.4.1	Boundary conditions for the amplitudes at the exit surface	166
6.4.2	Reflectivity	167
6.4.3	Properties of the rocking curves	168
6.5	Integrated intensity	170
7	Intensities of plane waves in the reflection geometry	173
7.1	Thick absorbing crystals	173
7.1.1	Reflectivity	173
7.1.2	Shape of the rocking curves	175
7.2	Standing waves	181
7.3	Thin crystals	185
7.3.1	Boundary conditions for the amplitudes	185
7.3.2	Reflectivity	186
8	Dynamical diffraction in highly asymmetric coplanar and non-coplanar geometries	189
8.1	Introduction	189
8.2	Diffraction at grazing incidence or grazing emergence	189
8.3	Deviation from Bragg's incidence of the middle of the reflection domain	192
8.3.1	Grazing incidence and Bragg geometry	192
8.3.2	Grazing incidence, Laue geometry	195
8.3.3	Grazing emergence	196
8.4	Variation of the Darwin width for a grazing incidence	197
8.5	Variation of the width of the diffracted beam for a grazing emergence	200
8.6	Equation of the dispersion surface	201
8.7	Relation with the traditional dynamical theory	206
8.8	Specularly and Bragg-reflected intensities	207
8.8.1	Boundary conditions for the amplitudes at the entrance surface	207
8.8.2	Specularly and Bragg-reflected intensities for a grazing incidence and the Bragg geometry (semi-infinite crystal)	210
8.8.3	Specularly and Bragg-reflected intensities for a grazing incidence and the Laue geometry	213
8.9	Grazing incidence diffraction (non-coplanar geometry)	213

8.9.1	Introduction	213
8.9.2	Three-dimensional representation of the dispersion surface	216
8.9.3	Tiepoints excited by the incident wave	216
8.9.4	Equation of the dispersion surface	223
8.9.5	Amplitudes of the waves	224
9	<i>n</i> -beam dynamical diffraction	225
9.1	Introduction	225
9.2	The general three-beam case	226
9.2.1	Renninger-scans	226
9.2.2	Fundamental equations of the dynamical theory	227
9.2.3	Solution in the general case	233
9.2.4	Energy flow	235
9.3	The three-beam coplanar case	236
9.4	Determination of phases using <i>n</i> -beam diffraction	236
9.5	The super-Bormann effect	242
9.5.1	Experimental evidence	242
9.5.2	Solution of the 111, $\bar{1}\bar{1}1$ case	243
9.5.3	Anomalous absorption coefficient	246
10	Spherical-wave dynamical theory: I. Kato's theory	249
10.1	Extension of the dynamical theory to any kind of incident wave	249
10.2	Fourier expansion of a spherical wave in plane waves	250
10.2.1	Principle of Kato's spherical-wave theory	250
10.2.2	The incident wave is a scalar wave	250
10.2.3	The incident wave is a vector wave	253
10.3	Direct integration in the transmission geometry	255
10.3.1	The geometrical conditions	255
10.3.2	Stationary phase method	257
10.3.3	Amplitude distribution on the exit surface—reflected wave	257
10.3.4	Amplitude distribution on the exit surface—refracted wave	260
10.4	Intensity distribution on the exit surface	260
10.5	Equal-intensity (Pendellösung) fringes	263
10.6	Integration by the stationary phase method	264
10.7	Integrated intensity	268
10.8	Influence of polarization	269
10.9	Bragg geometry	269
	Appendix: Geometrical interpretation of $\eta/\sqrt{S(\gamma_h) + \eta^2}$ in the transmission geometry	274

11	Spherical-wave dynamical theory: II. Takagi's theory	277
11.1	Introduction	277
11.2	Generalized fundamental equations	279
11.2.1	Modulated waves	279
11.2.2	Takagi's equations	280
11.2.3	Boundary conditions for the amplitudes at the entrance surface	283
11.3	Reduction of Takagi's equations in the plane-wave case	285
11.4	Absorbing crystals	286
11.5	Analytical resolution of Takagi's equations for perfect crystals	286
11.6	Analytical solution for a point source using the method of integral equations	287
11.6.1	Transmission geometry	288
11.6.2	Reflection geometry	290
11.7	Analytical resolution of Takagi's equations using the Riemann function	291
11.7.1	Hyperbolic nature of Takagi's equations	291
11.7.2	General expression of the reflected and refracted waves	292
11.7.3	Determination of the Riemann function	293
11.7.4	General solution of Takagi's equations	295
11.8	Analytical solution for an incident spherical wave using the method of Riemann functions	295
11.8.1	The incident wave is a point source located on the entrance surface	295
11.8.2	The incident wave is a point source located away from the entrance surface	296
11.8.3	Conservation of energy	298
	Appendix: Hyperbolic partial differential equations	299
	Characteristics	299
	Adjoint differential expression	301
12	Ray tracing in perfect crystals	304
12.1	Ray tracing	304
12.2	The structure of real waves	305
12.3	Wavepackets made of the superposition of separate plane waves	306
12.4	Wavepackets made of a continuous distribution of wavevectors	308
12.5	Group velocity and Poynting vector	310
12.6	Angular amplification	311
12.7	Intensity distribution along the base of the Borrmann triangle (transmission geometry)	317

12.8	Geometrical properties of wavefield trajectories within the Borrmann triangle	323
12.8.1	Wavefields propagating along the median, AE , of the Borrmann triangle	323
12.8.2	Properties of the trajectories of the two wavefields excited by a plane wave	323
12.9	Experimental proof of double refraction	324
12.10	Experimental observation of the separation of the wavefield paths	326
12.10.1	Experimental setup	326
12.10.2	Focalization of the various wavelengths	328
12.10.3	Separation of wavefield paths in the transmission case	329
12.10.4	Plane-wave Pendellösung	330
12.10.5	Application to the measurement of the index of refraction	332
12.11	Fresnel diffraction near the Bragg incidence	335
12.12	Ray tracing in finite crystals	339
12.12.1	Introduction	339
12.12.2	Bragg-Laue geometry—pseudo-plane waves	341
12.12.3	Bragg-Bragg geometry; multiple reflections of a pseudo-plane wave in thin crystals	343
12.12.4	Laue-Bragg geometry—Borrmann-Lehmann fringes	344
12.13	Coherence of extended, non-strictly monochromatic sources	349
III	Extension of the dynamical theory to deformed crystals	353
13	Ray tracing in slightly deformed crystals	355
13.1	X-ray propagation in deformed materials	355
13.1.1	The different degrees of deformation	355
13.1.2	Principle of ray theories for weak deformations	356
13.2	Effective misorientation	357
13.2.1	Local reciprocal lattice vector	357
13.2.2	Effective misorientation in direct space	359
13.2.3	Effective misorientation in reciprocal space	360
13.2.4	Strain gradient	362
13.3	Polarizability of a deformed crystal	363
13.4	The Eikonal approximation	363
13.4.1	Justification of the concept of local dispersion surface	363
13.4.2	Fermat's principle	365
13.5	Ray trajectories	368
13.5.1	Local dispersion surface	368
13.5.2	Local wavevectors	369
13.5.3	Differential equation of the wavefield trajectories	369

13.6	The case of a constant strain gradient	375
13.6.1	Equation of the ray trajectory with respect to the lattice planes	375
13.6.2	Ray trajectories in the transmission geometry	377
13.6.3	Pure bending	379
13.6.4	Temperature gradient	382
13.6.5	Ray trajectories in the reflection geometry	382
13.7	Diffracted intensities—plane-wave case	386
13.7.1	Zero absorption	386
13.7.2	Absorbing crystals (transmission geometry)	389
13.7.3	Expression of the diffracted intensities for a constant strain gradient	389
13.7.4	Discussion of the intensity distribution for a constant strain gradient	391
13.8	Diffracted intensities—spherical-wave case	395
13.8.1	Pendellösung in slightly deformed crystals	395
13.8.2	Phase of the refracted wave in a deformed crystal	397
13.8.3	Expression of the phase in terms of the coordinates in direct space	401
13.8.4	Shape of the Pendellösung fringes in a deformed crystal	403
14	Propagation of X-rays in highly deformed crystals	406
14.1	Introduction	406
14.2	Takagi's equations in a deformed crystal	406
14.3	Resolution of Takagi's equations in the deformed crystal case	409
14.3.1	Small deformations, limit of the validity of the Eikonal approximation	409
14.3.2	Analytical resolution of Takagi's equations	410
14.3.3	Numerical integration	415
14.3.4	Applications	420
14.4	Ray concept applied to highly distorted crystals	421
14.4.1	Generalization of the notion of wavefields, interbranch scattering	421
14.4.2	Example: X-ray propagation in a crystal with a concentration gradient (Keitel et al. 1999)	423
14.5	Statistical dynamical theories	426
14.5.1	Introduction	426
14.5.2	Principle of Kato's statistical dynamical theory	427
14.5.3	Experimental tests of the statistical dynamical theory	431
	Appendix: Resolution of Takagi's equations in the case of a constant strain gradient using Laplace transforms (Katagawa and Kato 1974)	432

IV Applications	435
15 X-ray optics	437
15.1 X-ray sources	437
15.1.1 X-ray tubes	437
15.1.2 Synchrotron radiation	439
15.2 <i>Flat monochromators</i>	445
15.2.1 Introduction	445
15.2.2 Monochromator crystals	446
15.2.3 Multiple-reflection monochromators	449
15.3 Applications of multiple-crystal arrangements to beam conditioning	456
15.3.1 Suppression of tails	456
15.3.2 Wavelength scanner	459
15.3.3 Production of beams with a very narrow angular spread	459
15.3.4 Harmonic suppression	461
15.3.5 Production of polarized radiation	467
15.4 Focusing optics	473
15.4.1 Introduction	473
15.4.2 Mirrors	474
15.4.3 Multilayers	476
15.4.4 Curved crystals	477
15.4.5 Fresnel zone plates	478
15.4.6 Bragg-Fresnel lenses	480
15.4.7 Refractive lenses	481
15.4.8 X-ray wave-guides	482
15.5 X-ray interferometers	483
15.5.1 Principle	483
15.5.2 Applications	486
15.6 Imaging with X-rays	489
15.6.1 Introduction	489
15.6.2 Phase contrast imaging	489
16 Location of atoms at surfaces and interfaces using X-ray standing waves	495
16.1 Principle	495
16.2 Theory	498
16.2.1 Fluorescence yield	498
16.2.2 Influence of thermal vibrations	502
16.3 Bulk crystals	502
16.3.1 Extinction effect	502
16.3.2 Determination of the polarity of heteropolar crystals	503
16.4 Solution to the surface registration problem	504

16.5	Thin films and buried interfaces	507
16.5.1	Simple model	507
16.5.2	Calculation of the standing pattern in an overlayer with the dynamical theory	509
16.6	Standing waves in deformed crystals	510
16.7	Standing waves due to specular reflection	511
17	X-ray diffraction topography	513
17.1	Introduction	513
17.2	Single-crystal reflection topography (Berg-Barrett technique)	514
17.2.1	Principle	514
17.2.2	Image formation	516
17.2.3	Penetration depth	518
17.2.4	Stereographic views	519
17.2.5	Applications	520
17.3	Single-crystal transmission topography	520
17.3.1	Early history	520
17.3.2	Principle of section topographs	523
17.3.3	Projection topographs	528
17.3.4	Dislocation images	538
17.3.5	Images of planar defects	551
17.3.6	Applications	560
17.4	Double- or multiple-crystal topography	564
17.4.1	Principle of double-crystal topography	564
17.4.2	Plane-wave topography	566
17.4.3	Synchrotron double-crystal topography	568
17.4.4	Mapping of distortions and of lattice parameter variations	568
17.4.5	Equal-strain or equal-lattice parameter contours	569
17.4.6	Double-crystal setting for high spatial resolution topography	569

Appendices

Appendix 1	Useful formulae	571
Appendix 2	The early days of dynamical theory	576
References		583
List of symbols		637
Index		641