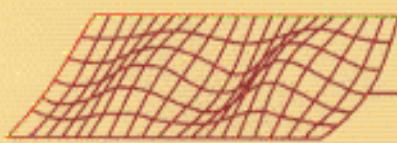


CONCEPTS
AND
APPLICATIONS
OF
FINITE ELEMENT
ANALYSIS



FOURTH EDITION

ROBERT D. COOK • DAVID S. MALKUS
MICHAEL E. PLESHA • ROBERT J. WITT

CONTENTS

NOTATION

Chapter 1 INTRODUCTION	1
1.1 Finite Element Analysis	1
1.2 Problem Classification, Modeling, and Discretization	3
1.3 Interpolation. Elements, Nodes, and D.O.F.	5
1.4 Example Applications. History of FEA	8
1.5 Solving a Problem by FEA	11
1.6 Learning and Using FEA	15
Analytical Problems	17
Chapter 2 ONE-DIMENSIONAL ELEMENTS AND COMPUTATIONAL PROCEDURES	19
2.1 Introduction	19
2.2 Bar Element	20
2.3 Beam Element	24
2.4 Bar and Beam Elements of Arbitrary Orientation	29
2.5 Assembly of Elements	32
2.6 Properties of Stiffness Matrices	36
2.7 Boundary Conditions	40
2.8 Exploiting Sparsity. Solving Equations	42
2.9 Mechanical Loads. Stresses	46
2.10 Thermal Loads. Stresses	52
2.11 Structural Symmetry	54
2.12 Review. Remarks Regarding Modeling	57
2.13 An Application	59
Analytical Problems	62
Computational Problems	74
Chapter 3 BASIC ELEMENTS	78
3.1 Preliminaries	78
3.2 Interpolation and Shape Functions	83
3.3 Formulas for Element Matrices	88
3.4 Linear Triangle (CST)	91
3.5 Quadratic Triangle (LST)	95
3.6 Bilinear Rectangle (Q4)	96
3.7 Quadratic Rectangle (Q8, Q9)	100
3.8 Rectangular Solid Elements	102
3.9 Choice of Interpolation Functions	104
3.10 Improved Triangles and Quadrilaterals	106

3.11	Nodal Loads	111
3.12	Stress Calculation	115
3.13	Nature of a Finite Element Solution	118
3.14	Example: A Simple Stress Concentration Problem	119
3.15	An Application with High Stress Gradient	121
	Analytical Problems	124
	Computational Problems	132

Chapter 4 FORMULATION TECHNIQUES: VARIATIONAL METHODS

136

4.1	Introduction	136
4.2	Principle of Stationary Potential Energy	137
4.3	Problems Having Many D.O.F.	140
4.4	Potential Energy of an Elastic Body	142
4.5	The Rayleigh-Ritz Method	146
4.6	Comments Regarding the Rayleigh-Ritz Method	149
4.7	Strong Form and Weak Form	151
4.8	Finite Element Form of the Rayleigh-Ritz Method	156
4.9	Convergence of Finite Element Solutions	161
4.10	Additional Formulations. Hybrid Elements	165
	Analytical Problems	171

Chapter 5 FORMULATION TECHNIQUES: GALERKIN AND OTHER WEIGHTED RESIDUAL METHODS

179

5.1	Galerkin Method	179
5.2	Methods of Weighted Residuals (MWR)	182
5.3	Galerkin Finite Element Method in One Dimension	186
5.4	Integration by Parts	191
5.5	Galerkin Finite Element Method in Two Dimensions	193
5.6	A Mixed Formulation	195
	Analytical Problems	198

Chapter 6 ISOPARAMETRIC ELEMENTS

202

6.1	Introduction	202
6.2	Bilinear Quadrilateral (Q4)	205
6.3	Quadrature: $[k]$ Obtained by Numerical Integration	209
6.4	Quadratic Quadrilaterals (Q8, Q9)	213
6.5	Hexahedral Isoparametric Elements	217
6.6	Incompatible Modes. Nodeless D.O.F.	219
6.7	Static Condensation	221
6.8	Choices in Numerical Integration	223
6.9	Load Considerations	227
6.10	Stress Calculation	230
6.11	Effect of Element Geometry	234
6.12	Validity of Isoparametric Elements	237
6.13	Patch Test	238

6.14	A 2D Application	240
6.15	A 3D Application	244
	Analytical Problems	247
	Computational Problems	255

Chapter 7 ISOPARAMETRIC TRIANGLES AND TETRAHEDRA **259**

7.1	Reference Coordinates. Shape Functions	259
7.2	Element Characteristic Matrices	262
7.3	Analytical Integration. Area and Volume Coordinates	264
7.4	Numerical Integration	266
	Analytical Problems	268

Chapter 8 COORDINATE TRANSFORMATION AND SELECTED ANALYSIS OPTIONS **271**

8.1	Transformation: Introduction and Vector Forms	271
8.2	Strain, Stress, and Material Property Transformation	273
8.3	Transformation of the Characteristic Matrix	275
8.4	Changing the Directions of Restraints	276
8.5	Connecting Dissimilar Elements. Rigid Elements	278
8.6	Higher Derivatives as Nodal D.O.F.	282
8.7	Fracture Mechanics. Singularity Elements	283
8.8	Elastic Foundations. Infinite Media	286
8.9	Structural Modification. Reanalysis	292
8.10	Tests of Element Quality	293
	Analytical Problems	295
	Computational Problems	299

Chapter 9 ERROR, ERROR ESTIMATION, AND CONVERGENCE **300**

9.1	Sources of Error	300
9.2	Ill-Conditioning	302
9.3	The Condition Number	306
9.4	Diagonal Decay Test	308
9.5	Residuals	309
9.6	Discretization Error. Convergence Rate	310
9.7	Multimesh Extrapolation	315
9.8	Mesh Revision Methods	318
9.9	Gradient (Stress) Recovery and Smoothing	320
9.10	<i>A-Posteriori</i> Error Estimate	326
9.11	Adaptive Meshing	329
	Analytical Problems	331
	Computational Problems	335

Chapter 10 MODELING CONSIDERATIONS AND SOFTWARE USE **336**

10.1	Introduction	336
10.2	Physical Behavior Versus Element Behavior	337
10.3	Element Shapes and Interconnection	340

10.4	Test Cases and Pilot Studies	342
10.5	Material Properties	344
10.6	Loads and Reactions	347
10.7	Connections in Structures	348
10.8	Boundary Conditions	352
10.9	Repetitive Symmetry	354
10.10	Stress Concentrations. Submodels	356
10.11	Substructures	358
10.12	Planning an Analysis	360
10.13	Common Mistakes	363
10.14	Checking the Model	365
10.15	Critique of Computed Results	366
10.16	Design Optimization	369
10.17	Software	370
10.18	Concluding Remarks	371
	Analytical Problems	371
	Computational Problems	372

Chapter 11 FINITE ELEMENTS IN STRUCTURAL DYNAMICS AND VIBRATIONS

373

11.1	Introduction	373
11.2	Dynamic Equations. Mass and Damping Matrices	374
11.3	Mass Matrices: Consistent, Diagonal, and Other	377
11.4	Natural Frequencies and Modes	383
11.5	Damping	388
11.6	Reduction of the Number of D.O.F.	390
11.7	Response History: Modal Methods	394
11.8	Response History: Ritz Vectors	398
11.9	Component Mode Synthesis (CMS)	400
11.10	Harmonic Response	405
11.11	Response History: Direct Integration Methods	407
11.12	Explicit Direct Integration	409
11.13	Implicit Direct Integration	416
11.14	Direct Integration: Stability and Accuracy Analysis	421
11.15	Analysis by Response Spectra	426
11.16	Remarks. Modeling Considerations	429
11.17	An Application: Vibration and Harmonic Response	436
11.18	An Application: Response History	439
	Analytical Problems	444
	Computational Problems	451

Chapter 12 HEAT TRANSFER AND SELECTED FLUID PROBLEMS **454**

12.1	Heat Transfer: Introduction	454
12.2	Finite Element Formulation	459
12.3	Radiation. Nonlinear Heat Transfer Problems	462
12.4	Transient Thermal Analysis	464
12.5	Modeling Considerations. Remarks	467

12.6	An Application	469
12.7	Acoustic Frequencies and Modes	474
12.8	Fluid-Structure Interaction	477
12.9	Plane Incompressible Irrotational Flow	480
	Analytical Problems	482
	Computational Problems	486
Chapter 13 CONSTRAINTS: PENALTY FORMS, LOCKING, AND CONSTRAINT COUNTING		489
13.1	Explicit Constraints. Transformation Equations	489
13.2	Lagrange Multipliers to Enforce Constraints	492
13.3	Penalty Functions to Enforce Constraints	493
13.4	Implicit Penalty Constraints and Locking	495
13.5	Constraint Counting	499
13.6	Remarks About Techniques for Incompressible Media	502
	Analytical Problems	504
Chapter 14 SOLIDS OF REVOLUTION		508
14.1	Introduction. Elasticity Relations for Axial Symmetry	508
14.2	Axisymmetric Solid Elements	510
14.3	An Application	512
14.4	Loads Without Axial Symmetry: Introduction	516
14.5	Loads Without Axial Symmetry: Some Details of FEA	521
	Analytical Problems	524
	Computational Problems	527
Chapter 15 PLATE BENDING		530
15.1	Introduction. Plate Behavior	530
15.2	C^1 (Kirchhoff) Plate Elements	536
15.3	C^0 (Mindlin) Plate Elements	542
15.4	Mindlin Beam. More Devices for C^0 Plate Elements	547
15.5	Boundary Conditions. Test Problems	551
15.6	An Application	553
	Analytical Problems	556
	Computational Problems	559
Chapter 16 SHELLS		561
16.1	Introduction	561
16.2	Circular Arches and Arch Elements	563
16.3	Shells of Revolution	570
16.4	General Shells: Three- and Four-Node Elements	574
16.5	General Shells: Curved Isoparametric Elements	578
16.6	Test Cases. Remarks	583
16.7	An Axisymmetric Shell Application	586
	Analytical Problems	588
	Computational Problems	591

Chapter 17 NONLINEARITY: AN INTRODUCTION	595
17.1 Nonlinear Problems 595	
17.2 Some Solution Methods 596	
17.3 Plasticity: Introduction 602	
17.4 Plasticity: General Formulation for Small Strains 606	
17.5 Plasticity: Formulation for Von Mises Theory 609	
17.6 Plasticity: Some Computational Procedures 612	
17.7 Nonlinear Dynamic Problems 616	
17.8 Problems of Gaps and Contact 619	
17.9 Geometric Nonlinearity 621	
17.10 Modeling Considerations, Remarks 626	
Analytical Problems 630	
Computational Problems 636	
Chapter 18 STRESS STIFFNESS AND BUCKLING	639
18.1 Introduction, Energy Considerations 639	
18.2 Bar and Beam Elements 642	
18.3 Plate Elements 645	
18.4 A General Formulation 646	
18.5 Calculation of Buckling Loads 648	
18.6 Remarks on Stress Stiffness and Its Uses 650	
18.7 Remarks and Examples 653	
Analytical Problems 656	
Computational Problems 661	
Appendix A MATRICES: SELECTED DEFINITIONS AND MANIPULATIONS	663
Appendix B SIMULTANEOUS ALGEBRAIC EQUATIONS	668
B.1 Overview 668	
B.2 Direct Solvers 668	
B.3 Iterative Solvers 671	
Appendix C EIGENVALUES AND EIGENVECTORS	675
C.1 Overview 675	
C.2 The Standard Eigenproblem 675	
C.3 The General Eigenproblem 676	
C.4 Solution Algorithms 679	
REFERENCES	682
INDEX	711