

John M. Howie

Real Analysis

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$\int_K \nabla F dV = \int_{\partial K} F \cdot \vec{n} d\sigma \longleftrightarrow \int_K dv = \int_{\partial K} \omega$$

$$\sim (P \cdot Q) \equiv \sim P \vee \sim Q, \sim (P \vee Q) \equiv \sim P \cdot \sim Q$$

$$|\langle \chi, \gamma \rangle| \leq \| \chi \| \| \gamma \|$$

$$\delta_f = \frac{1}{|G|} \sum_{g \in G} \chi_f(g) \overline{\chi_f(g)} = \frac{1}{|G|} \sum_{g \in G} |\chi_f(g)|^2$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



Springer

$$\int_a^b f(t) dt = F(b) - F(a)$$



SPRINGER



UNDERGRADUATE



MATHEMATICS



SERIES

Contents

1. Introductory Ideas	1
1.1 Foreword for the Student: Is Analysis Necessary?	1
1.2 The Concept of Number	3
1.3 The Language of Set Theory	4
1.4 Real Numbers	7
1.5 Induction	12
1.6 Inequalities	18
2. Sequences and Series	27
2.1 Sequences	27
2.2 Sums, Products and Quotients	33
2.3 Monotonic Sequences	37
2.4 Cauchy Sequences	42
2.5 Series	47
2.6 The Comparison Test	50
2.7 Series of Positive and Negative Terms	58
3. Functions and Continuity	63
3.1 Functions, Graphs	63
3.2 Sums, Products, Compositions; Polynomial and Rational Functions	66
3.3 Circular Functions	70
3.4 Limits	73
3.5 Continuity	81
3.6 Uniform Continuity	90
3.7 Inverse Functions	94

4. Differentiation	99
4.1 The Derivative	99
4.2 The Mean Value Theorems	105
4.3 Inverse Functions	110
4.4 Higher Derivatives	113
4.5 Taylor's Theorem	116
5. Integration	119
5.1 The Riemann Integral	119
5.2 Classes of Integrable Functions	126
5.3 Properties of Integrals	131
5.4 The Fundamental Theorem	138
5.5 Techniques of Integration	143
5.6 Improper Integrals of the First Kind	150
5.7 Improper Integrals of the Second Kind	158
6. The Logarithmic and Exponential Functions	165
6.1 A Function Defined by an Integral	165
6.2 The Inverse Function	168
6.3 Further Properties of the Exponential and Logarithmic Functions	176
7. Sequences and Series of Functions	181
7.1 Uniform Convergence	181
7.2 Uniform Convergence of Series	192
7.3 Power Series	201
8. The Circular Functions	217
8.1 Definitions and Elementary Properties	217
8.2 Length	220
9. Miscellaneous Examples	229
9.1 Wallis's Formula	229
9.2 Stirling's Formula	230
9.3 A Continuous, Nowhere Differentiable Function	234
Solutions to Exercises	237
The Greek Alphabet	269
Bibliography	271
Index	273