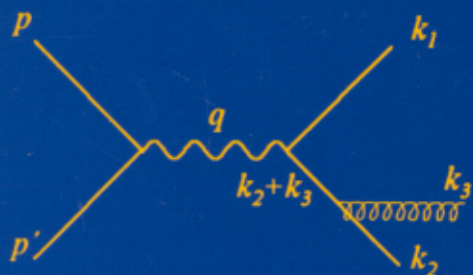


Gauge Theory of Elementary Particle Physics

Problems and Solutions

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OXFORD

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