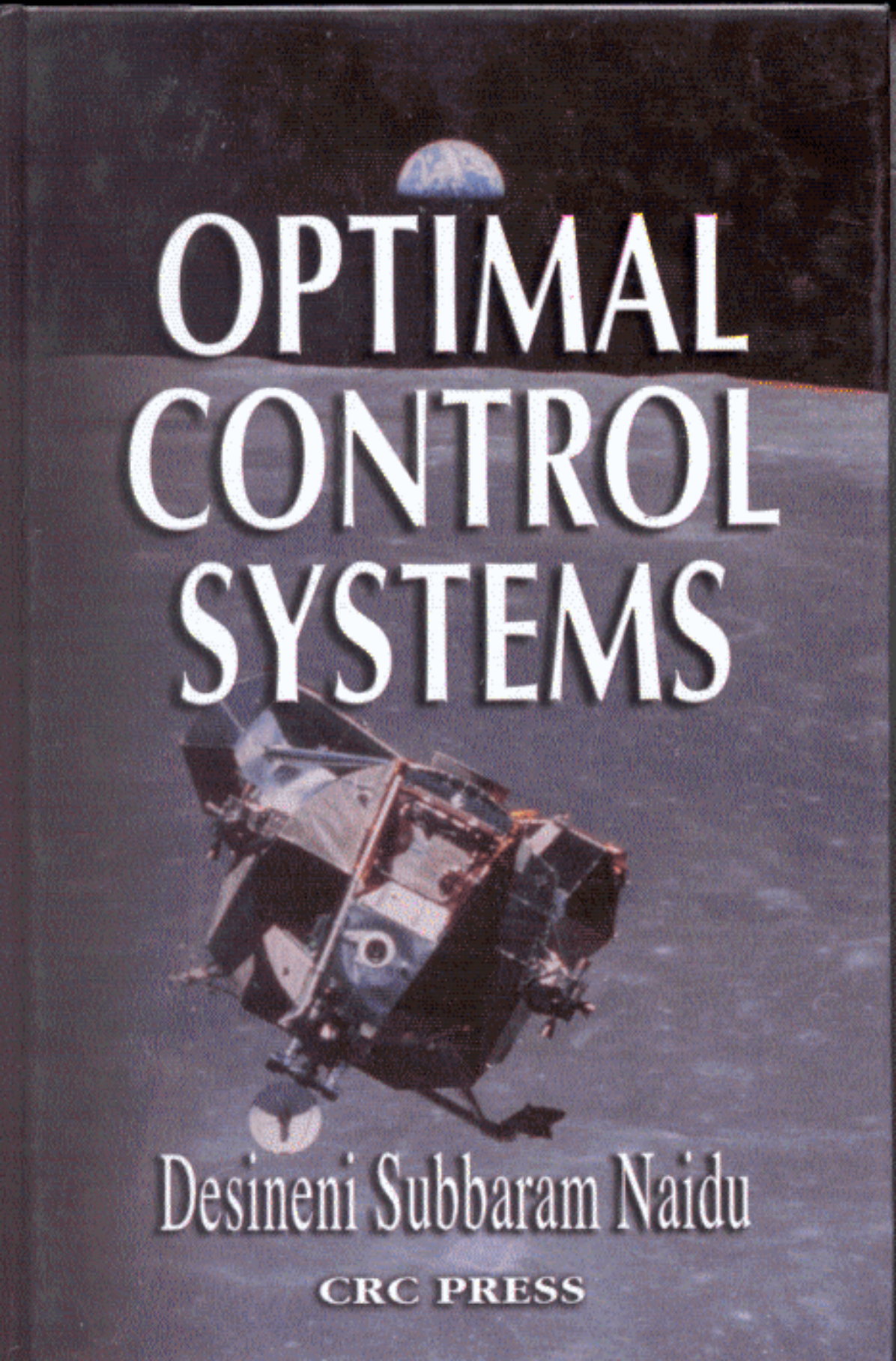


The background of the cover is a dark, grainy image of space. At the top, a small, curved horizon of the Earth is visible against the blackness of space. Below this, a satellite is shown in a three-quarter view, tilted diagonally. The satellite has a complex, boxy structure with various panels, antennas, and a large circular dish-like component on one side. It appears to be in orbit.

OPTIMAL CONTROL SYSTEMS

Desineni Subbaram Naidu

CRC PRESS

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