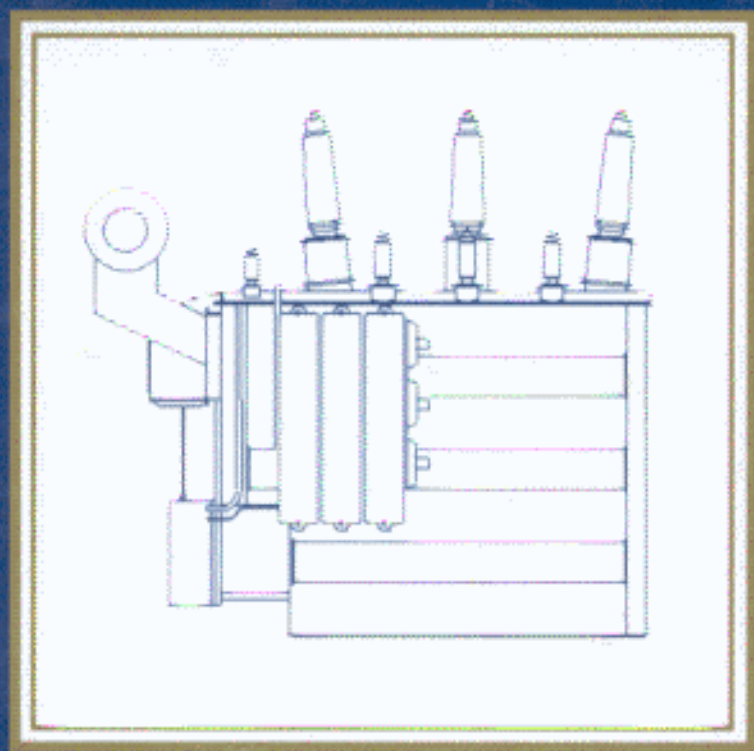


# TRANSFORMER DESIGN PRINCIPLES

*With Applications to  
Core-Form Power Transformers*



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