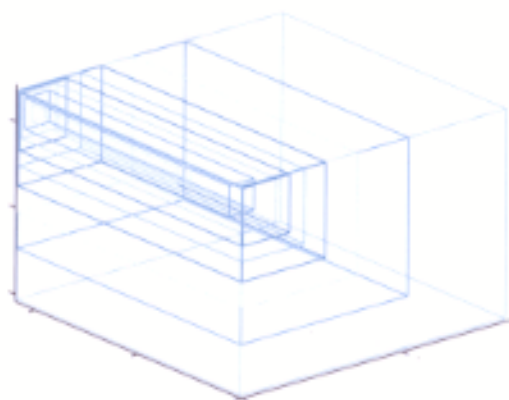
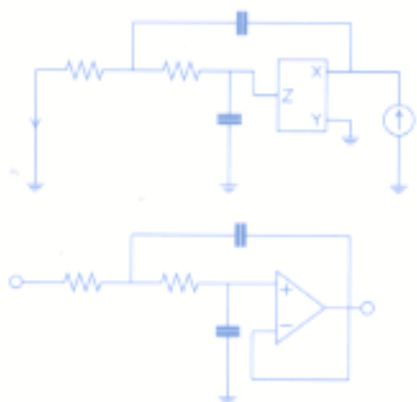
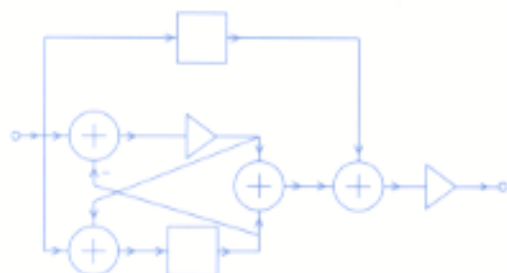
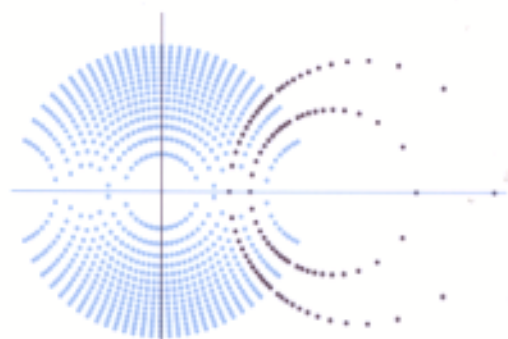


FILTER DESIGN FOR SIGNAL PROCESSING

Using MATLAB® and Mathematica®



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