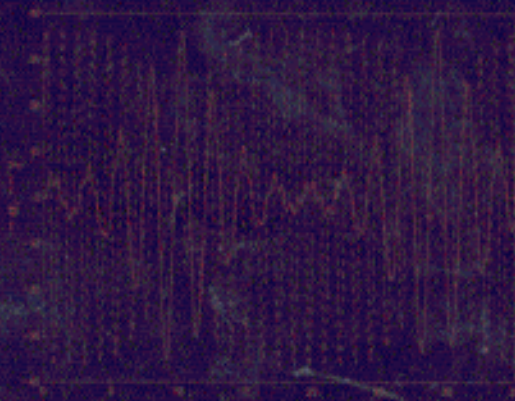


FUNDAMENTALS OF ENGINEERING NUMERICAL ANALYSIS

PARVIZ MOIN



$$I_0 \ddot{\alpha} + a m_e g \sin \alpha + b m_e [C \dot{\gamma} + S \dot{\gamma}^2] = 0$$
$$I_0 \ddot{\gamma} + c m_e g \sin \gamma + b m_e [C \dot{\alpha} + S \dot{\alpha}^2] = 0$$



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