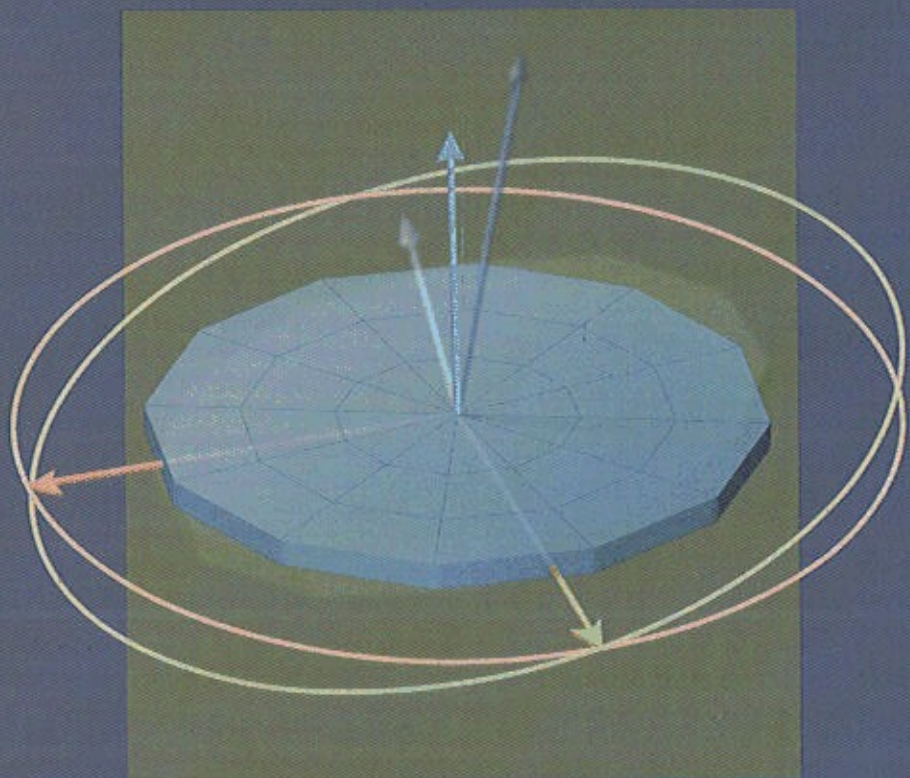


An Introduction to
**COMPUTER
SIMULATION
METHODS**
APPLICATIONS TO PHYSICAL SYSTEMS



THIRD EDITION
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