

Linear Models for the Prediction of Animal Breeding Values, 2nd Edition

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$$(\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Z} + \mathbf{G}^{-1}\alpha_{\text{anim}})\hat{\mathbf{a}}_{\text{anim}} = 2\mathbf{G}^{-1}\alpha_{\text{par}}(\mathbf{PA}) + (\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Z})\mathbf{YD} + 0.5\mathbf{G}$$

$$\begin{pmatrix} \hat{a}_{s1} \\ \hat{a}_{s2} \end{pmatrix} = \mathbf{W}_1 \begin{pmatrix} PA_{s1} \\ PA_{s2} \end{pmatrix} + \mathbf{W}_2 \begin{pmatrix} YD_{s1} \\ YD_{s2} \end{pmatrix} = \mathbf{W}_1 \begin{pmatrix} 0.099 \\ 0.1735 \end{pmatrix} + \mathbf{W}_2 \begin{pmatrix} 0.639 \\ 0.700 \end{pmatrix}$$



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$$\mathbf{A}^{-1}_i = \begin{bmatrix} \mathbf{A}^{-1}_{i-1} & 0 \\ 0 & 0 \end{bmatrix} + (a_{ii} - s'_i \mathbf{A}_{i-1} s_i)^{-1} \begin{bmatrix} s_i s'_i & -s_i \\ -s'_i & 1 \end{bmatrix}$$

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