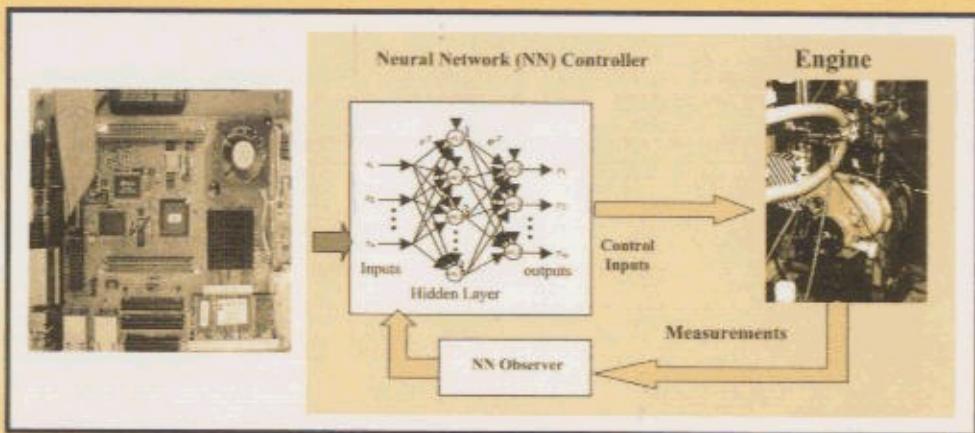


**Control Engineering Series**

# **Neural Network Control of Nonlinear Discrete-Time Systems**



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**Taylor & Francis**  
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