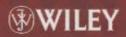
Dealing with

Texture

Maria Petrou Pedro García Sevilla



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