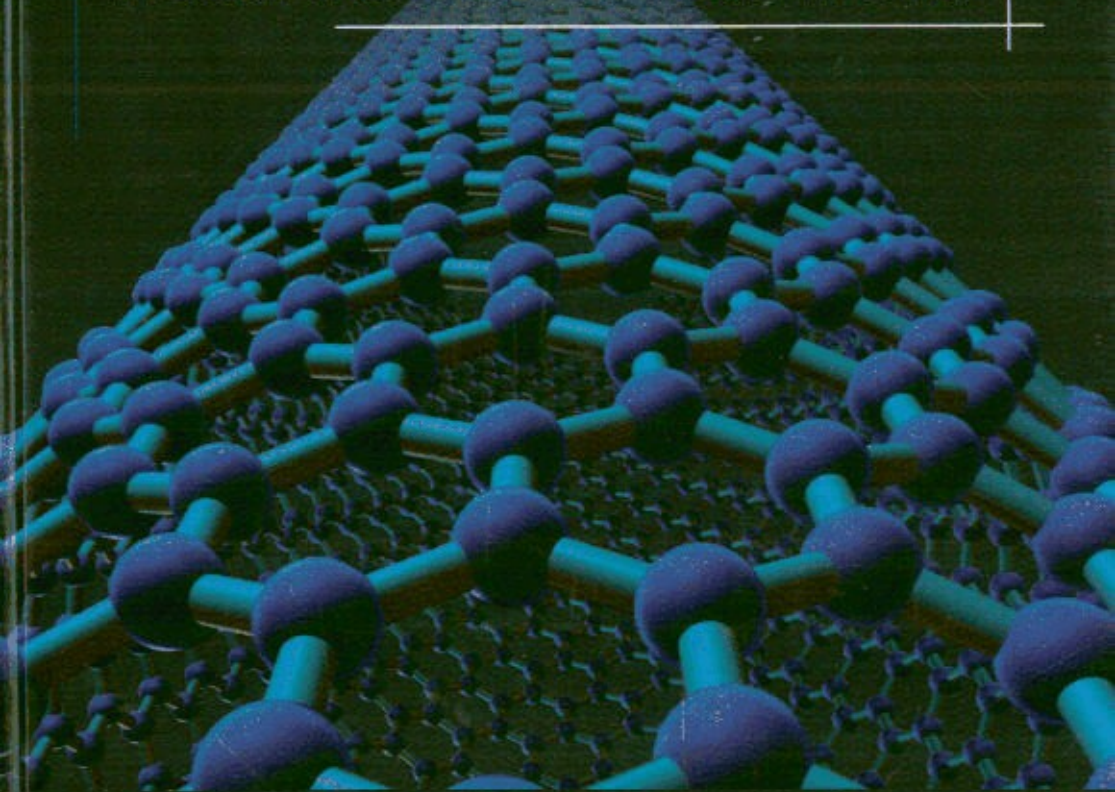


McGraw-Hill Nanoscience and Technology Series

# MECHANICAL DESIGN OF MICRORESONATORS

MODELING AND APPLICATIONS



NICOLAE O. LOBONTIU

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