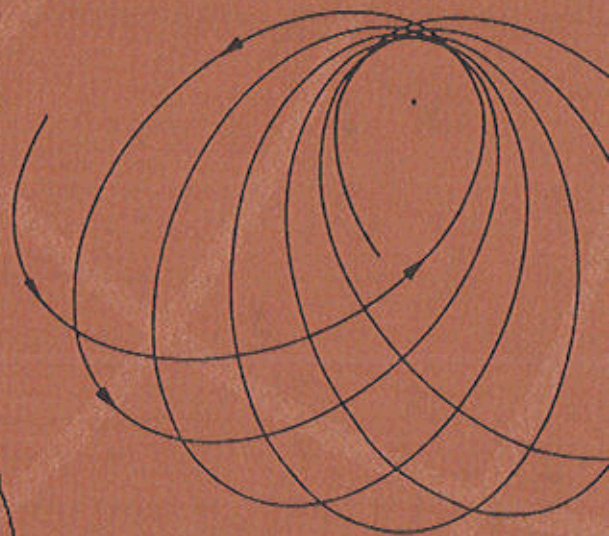
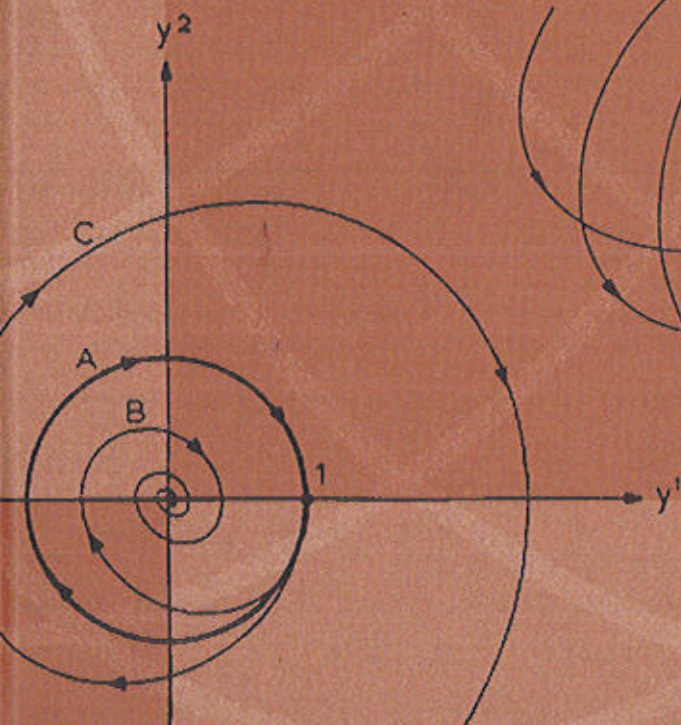



Florian Scheck

# Mechanics

From Newton's Laws  
to Deterministic Chaos

Fourth Edition



 Springer

# Contents

<b>1. Elementary Newtonian Mechanics</b> .....	1
1.1 Newton's Laws (1687) and Their Interpretation .....	1
1.2 Uniform Rectilinear Motion and Inertial Systems .....	4
1.3 Inertial Frames in Relative Motion .....	6
1.4 Momentum and Force .....	6
1.5 Typical Forces. A Remark About Units .....	8
1.6 Space, Time, and Forces .....	10
1.7 The Two-Body System with Internal Forces .....	11
1.7.1 Center-of-Mass and Relative Motion .....	11
1.7.2 Example: The Gravitational Force Between Two Celestial Bodies (Kepler's Problem) .....	13
1.7.3 Center-of-Mass and Relative Momentum in the Two-Body System .....	19
1.8 Systems of Finitely Many Particles .....	20
1.9 The Principle of Center-of-Mass Motion .....	21
1.10 The Principle of Angular-Momentum Conservation .....	21
1.11 The Principle of Energy Conservation .....	22
1.12 The Closed $n$ -Particle System .....	23
1.13 Galilei Transformations .....	24
1.14 Space and Time with Galilei Invariance .....	27
1.15 Conservative Force Fields .....	29
1.16 One-Dimensional Motion of a Point Particle .....	32
1.17 Examples of Motion in One Dimension .....	33
1.17.1 The Harmonic Oscillator .....	33
1.17.2 The Planar Mathematical Pendulum .....	35
1.18 Phase Space for the $n$ -Particle System (in $\mathbb{R}^3$ ) .....	37
1.19 Existence and Uniqueness of the Solutions of $\dot{x} = F(x, t)$ .....	38
1.20 Physical Consequences of the Existence and Uniqueness Theorem .....	39
1.21 Linear Systems .....	41
1.21.1 Linear, Homogeneous Systems .....	41
1.21.2 Linear, Inhomogeneous Systems .....	42
1.22 Integrating One-Dimensional Equations of Motion .....	42
1.23 Example: The Planar Pendulum for Arbitrary Deviations from the Vertical .....	44

1.24	Example: The Two-Body System with a Central Force	47
1.25	Rotating Reference Systems: Coriolis and Centrifugal Forces	54
1.26	Examples of Rotating Reference Systems	55
1.27	Scattering of Two Particles that Interact via a Central Force:	
	Kinematics	63
1.28	Two-Particle Scattering with a Central Force: Dynamics	67
1.29	Example: Coulomb Scattering of Two Particles	
	with Equal Mass and Charge	71
1.30	Mechanical Bodies of Finite Extension	75
1.31	Time Averages and the Virial Theorem	79
	Appendix: Practical Examples	81
<b>2.</b>	<b>The Principles of Canonical Mechanics</b>	<b>87</b>
2.1	Constraints and Generalized Coordinates	87
	2.1.1 Definition of Constraints	87
	2.1.2 Generalized Coordinates	89
2.2	D'Alembert's Principle	89
	2.2.1 Definition of Virtual Displacements	89
	2.2.2 The Static Case	90
	2.2.3 The Dynamical Case	90
2.3	Lagrange's Equations	92
2.4	Examples of the Use of Lagrange's Equations	93
2.5	A Digression on Variational Principles	95
2.6	Hamilton's Variational Principle (1834)	98
2.7	The Euler-Lagrange Equations	98
2.8	Further Examples of the Use of Lagrange's Equations	99
2.9	A Remark About Nonuniqueness of the Lagrangian Function	101
2.10	Gauge Transformations of the Lagrangian Function	102
2.11	Admissible Transformations of the Generalized Coordinates	103
2.12	The Hamiltonian Function and Its Relation	
	to the Lagrangian Function $L$	104
2.13	The Legendre Transformation for the Case of One Variable	105
2.14	The Legendre Transformation for the Case of Several Variables	107
2.15	Canonical Systems	108
2.16	Examples of Canonical Systems	109
2.17	The Variational Principle Applied to the Hamiltonian Function	111
2.18	Symmetries and Conservation Laws	112
2.19	Noether's Theorem	113
2.20	The Generator for Infinitesimal Rotations About an Axis	115
2.21	More About the Rotation Group	117
2.22	Infinitesimal Rotations and Their Generators	119
2.23	Canonical Transformations	121
2.24	Examples of Canonical Transformations	125
2.25	The Structure of the Canonical Equations	126
2.26	Example: Linear Autonomous Systems in One Dimension	127

2.27	Canonical Transformations in Compact Notation	129
2.28	On the Symplectic Structure of Phase Space	131
2.29	Liouville's Theorem	134
2.29.1	The Local Form	135
2.29.2	The Global Form	136
2.30	Examples for the Use of Liouville's Theorem	137
2.31	Poisson Brackets	140
2.32	Properties of Poisson Brackets	143
2.33	Infinitesimal Canonical Transformations	145
2.34	Integrals of the Motion	146
2.35	The Hamilton–Jacobi Differential Equation	149
2.36	Examples for the Use of the Hamilton–Jacobi Equation	150
2.37	The Hamilton–Jacobi Equation and Integrable Systems	154
2.37.1	Local Rectification of Hamiltonian Systems	154
2.37.2	Integrable Systems	158
2.37.3	Angle and Action Variables	163
2.38	Perturbing Quasiperiodic Hamiltonian Systems	164
2.39	Autonomous, Nondegenerate Hamiltonian Systems in the Neighborhood of Integrable Systems	167
2.40	Examples. The Averaging Principle	168
2.40.1	The Anharmonic Oscillator	168
2.40.2	Averaging of Perturbations	170
2.41	Generalized Theorem of Noether	172
	Appendix: Practical Examples	180
<b>3.</b>	<b>The Mechanics of Rigid Bodies</b>	<b>185</b>
3.1	Definition of Rigid Body	185
3.2	Infinitesimal Displacement of a Rigid Body	187
3.3	Kinetic Energy and the Inertia Tensor	189
3.4	Properties of the Inertia Tensor	191
3.5	Steiner's Theorem	195
3.6	Examples of the Use of Steiner's Theorem	196
3.7	Angular Momentum of a Rigid Body	201
3.8	Force-Free Motion of Rigid Bodies	203
3.9	Another Parametrization of Rotations: The Euler Angles	205
3.10	Definition of Eulerian Angles	207
3.11	Equations of Motion of Rigid Bodies	208
3.12	Euler's Equations of Motion	211
3.13	Euler's Equations Applied to a Force-Free Top	214
3.14	The Motion of a Free Top and Geometric Constructions	218
3.15	The Rigid Body in the Framework of Canonical Mechanics	221
3.16	Example: The Symmetric Children's Top in a Gravitational Field	225
3.17	More About the Spinning Top	227
3.18	Spherical Top with Friction: The "Tippe Top"	229

3.18.1	Conservation Law and Energy Considerations	230
3.18.2	Equations of Motion and Solutions with Constant Energy	232
Appendix:	Practical Examples	236
<b>4.</b>	<b>Relativistic Mechanics</b>	239
4.1	Failures of Nonrelativistic Mechanics	240
4.2	Constancy of the Speed of Light	243
4.3	The Lorentz Transformations	244
4.4	Analysis of Lorentz and Poincaré Transformations	250
4.4.1	Rotations and Special Lorentz Transformations ("Boosts")	252
4.4.2	Interpretation of Special Lorentz Transformations	256
4.5	Decomposition of Lorentz Transformations into Their Components	257
4.5.1	Proposition on Orthochronous, Proper Lorentz Transformations	257
4.5.2	Corollary of the Decomposition Theorem and Some Consequences	259
4.6	Addition of Relativistic Velocities	262
4.7	Galilean and Lorentzian Space–Time Manifolds	264
4.8	Orbital Curves and Proper Time	268
4.9	Relativistic Dynamics	270
4.9.1	Newton’s Equation	270
4.9.2	The Energy–Momentum Vector	272
4.9.3	The Lorentz Force	275
4.10	Time Dilatation and Scale Contraction	277
4.11	More About the Motion of Free Particles	279
4.12	The Conformal Group	282
<b>5.</b>	<b>Geometric Aspects of Mechanics</b>	283
5.1	Manifolds of Generalized Coordinates	284
5.2	Differentiable Manifolds	287
5.2.1	The Euclidean Space $\mathbb{R}^n$	287
5.2.2	Smooth or Differentiable Manifolds	289
5.2.3	Examples of Smooth Manifolds	291
5.3	Geometrical Objects on Manifolds	295
5.3.1	Functions and Curves on Manifolds	296
5.3.2	Tangent Vectors on a Smooth Manifold	298
5.3.3	The Tangent Bundle of a Manifold	300
5.3.4	Vector Fields on Smooth Manifolds	301
5.3.5	Exterior Forms	305
5.4	Calculus on Manifolds	307
5.4.1	Differentiable Mappings of Manifolds	307
5.4.2	Integral Curves of Vector Fields	309

5.4.3	Exterior Product of One-Forms .....	311
5.4.4	The Exterior Derivative .....	313
5.4.5	Exterior Derivative and Vectors in $\mathbb{R}^3$ .....	315
5.5	Hamilton–Jacobi and Lagrangian Mechanics .....	317
5.5.1	Coordinate Manifold $Q$ , Velocity Space $TQ$ , and Phase Space $T^*Q$ .....	317
5.5.2	The Canonical One-Form on Phase Space .....	321
5.5.3	The Canonical, Symplectic Two-Form on $M$ .....	324
5.5.4	Symplectic Two-Form and Darboux’s Theorem .....	326
5.5.5	The Canonical Equations .....	329
5.5.6	The Poisson Bracket .....	332
5.5.7	Time-Dependent Hamiltonian Systems .....	335
5.6	Lagrangian Mechanics and Lagrange Equations .....	337
5.6.1	The Relation Between the Two Formulations of Mechanics .....	337
5.6.2	The Lagrangian Two-Form .....	339
5.6.3	Energy Function on $TQ$ and Lagrangian Vector Field ...	340
5.6.4	Vector Fields on Velocity Space $TQ$ and Lagrange Equations .....	342
5.6.5	The Legendre Transformation and the Correspondence of Lagrangian and Hamiltonian Functions .....	344
5.7	Riemannian Manifolds in Mechanics .....	347
5.7.1	Affine Connection and Parallel Transport .....	348
5.7.2	Parallel Vector Fields and Geodesics .....	350
5.7.3	Geodesics as Solutions of Euler–Lagrange Equations ...	351
5.7.4	Example: Force-Free Asymmetric Top .....	352
<b>6.</b>	<b>Stability and Chaos .....</b>	<b>355</b>
6.1	Qualitative Dynamics .....	355
6.2	Vector Fields as Dynamical Systems .....	356
6.2.1	Some Definitions of Vector Fields and Their Integral Curves .....	358
6.2.2	Equilibrium Positions and Linearization of Vector Fields .....	360
6.2.3	Stability of Equilibrium Positions .....	363
6.2.4	Critical Points of Hamiltonian Vector Fields .....	367
6.2.5	Stability and Instability of the Free Top .....	369
6.3	Long-Term Behavior of Dynamical Flows and Dependence on External Parameters .....	371
6.3.1	Flows in Phase Space .....	372
6.3.2	More General Criteria for Stability .....	373
6.3.3	Attractors .....	376
6.3.4	The Poincaré Mapping .....	380
6.3.5	Bifurcations of Flows at Critical Points .....	384
6.3.6	Bifurcations of Periodic Orbits .....	388

6.4	Deterministic Chaos .....	390
6.4.1	Iterative Mappings in One Dimension .....	390
6.4.2	Qualitative Definitions of Deterministic Chaos .....	392
6.4.3	An Example: The Logistic Equation .....	396
6.5	Quantitative Measures of Deterministic Chaos .....	401
6.5.1	Routes to Chaos .....	401
6.5.2	Liapunov Characteristic Exponents .....	405
6.5.3	Strange Attractors .....	407
6.6	Chaotic Motions in Celestial Mechanics .....	409
6.6.1	Rotational Dynamics of Planetary Satellites .....	409
6.6.2	Orbital Dynamics of Asteroids with Chaotic Behavior ...	415
<b>7.</b>	<b>Continuous Systems</b> .....	<b>419</b>
7.1	Discrete and Continuous Systems .....	419
7.2	Transition to the Continuous System .....	423
7.3	Hamilton's Variational Principle for Continuous Systems .....	425
7.4	Canonically Conjugate Momentum and Hamiltonian Density .....	427
7.5	Example: The Pendulum Chain .....	428
7.6	Comments and Outlook .....	432
<b>Exercises</b>	.....	<b>437</b>
	Chapter 1: Elementary Newtonian Mechanics .....	437
	Chapter 2: The Principles of Canonical Mechanics .....	444
	Chapter 3: The Mechanics of Rigid Bodies .....	452
	Chapter 4: Relativistic Mechanics .....	455
	Chapter 5: Geometric Aspects of Mechanics .....	458
	Chapter 6: Stability and Chaos .....	460
<b>Solution of Exercises</b>	.....	<b>463</b>
	Chapter 1: Elementary Newtonian Mechanics .....	463
	Chapter 2: The Principles of Canonical Mechanics .....	478
	Chapter 3: The Mechanics of Rigid Bodies .....	497
	Chapter 4: Relativistic Mechanics .....	505
	Chapter 5: Geometric Aspects of Mechanics .....	517
	Chapter 6: Stability and Chaos .....	522
<b>Author Index</b>	.....	<b>541</b>
<b>Subject Index</b>	.....	<b>543</b>