

The background of the cover is a complex, abstract visualization. It features a central cluster of glowing orange and red spheres, from which numerous thin, white and blue lines radiate outwards, creating a starburst or network-like effect. The overall color palette transitions from dark blue and black at the top and bottom to bright orange and yellow in the middle, suggesting a sunset or a high-energy physical process.

Computation in  
**M**ODERN  
Physics  
Third Edition

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# Contents

<i>List of Figures</i>	ix
<i>Preface</i>	xi
1. Integration	1
1.1 Classical Quadrature	1
1.2 Orthogonal Polynomials	10
1.2.1 Orthogonal Polynomials in the Interval $-1 \leq x \leq 1$	10
1.2.2 General Orthogonal Polynomials	13
1.3 Gaussian Integration	14
1.3.1 Gauss-Legendre Integration	16
1.3.2 Gauss-Laguerre Integration	16
1.4 Special Integration Schemes	19
1.5 Principal Value Integrals	20
2. Introduction to Monte Carlo	27
2.1 Preliminary Notions - - Calculating $\pi$	27
2.2 Evaluation of Integrals by Monte Carlo	29
2.3 Techniques for Direct Sampling	32
2.3.1 Cumulative Probability Distributions	33
2.3.2 The Characteristic Function $\phi(t)$	33
2.3.3 The Fundamental Theorem of Sampling	34
2.3.4 Sampling Monomials $0 \leq x \leq 1$	35
2.3.5 Sampling Functions $0 \leq x \leq \infty$	37
2.3.5.1 The Exponential Function	37
2.3.5.2 Other Algebraically Invertible Functions	37
2.3.5.3 Sampling a Gaussian Distribution	40
2.3.6 Brute-force Inversion of $F(x)$	41
2.3.7 The Rejection Technique	42
2.3.8 Sums of Random Variables	43
2.3.9 Selection on the Random Variables	44
2.3.10 The Sum of Probability Distribution Functions	47
2.3.10.1 Special Cases	49
2.4 The Metropolis Algorithm	50
2.4.1 The Method Itself	50
2.4.2 Why It Works	53

2.4.3	Comments on the Algorithm . . . . .	54
3.	Differential Methods . . . . .	61
3.1	Difference Schemes . . . . .	61
3.1.1	Elementary Considerations . . . . .	61
3.1.2	The General Case . . . . .	62
3.2	Simple Differential Equations . . . . .	64
3.3	Modeling with Differential Equations . . . . .	68
4.	Computers for Physicists . . . . .	75
4.1	Fundamentals . . . . .	76
4.1.1	Representation of Negative Numbers . . . . .	77
4.1.2	Logical Operations . . . . .	79
4.1.3	Integer Formats . . . . .	80
4.1.3.1	Fixed Point Lengths . . . . .	80
4.1.4	Floating Point Formats . . . . .	81
4.1.5	Some Practical Conclusions . . . . .	83
4.2	The i80X86 Series . . . . .	84
4.2.1	The Stack . . . . .	84
4.2.2	Memory Addressing . . . . .	85
4.2.3	Internal Registers of the CPU . . . . .	86
4.2.4	Instructions . . . . .	87
4.2.5	A Sample Program . . . . .	92
4.2.6	The Floating Point Co-processor i8087 . . . . .	94
4.2.7	Two Important Bottlenecks . . . . .	95
4.3	Cray-1 S Architecture . . . . .	95
4.3.1	Vector Operations and Chaining . . . . .	96
4.3.2	Coding for Maximum Speed . . . . .	97
4.4	Intel i860 Architecture . . . . .	98
4.5	Multi-Processor Computer Systems . . . . .	102
4.5.1	Amdahl's Law . . . . .	102
4.5.2	Difficulties . . . . .	103
4.5.3	One Practical Solution: Beowulf Clusters . . . . .	103
4.5.4	Algorithm types . . . . .	105
4.5.4.1	"100%" Algorithms . . . . .	105
4.5.4.2	Semi-efficient Algorithms . . . . .	106
4.5.4.3	Costly algorithms . . . . .	106
4.6	A Parallel Recursive Algorithm . . . . .	108
5.	Linear Algebra . . . . .	115
5.1	$\chi^2$ Analysis . . . . .	115
5.2	Solution of Linear Equations . . . . .	117
5.2.1	Gaussian Elimination . . . . .	117
5.2.2	LU Reductions . . . . .	120
5.2.3	Crout's LU Reduction . . . . .	122
5.2.4	The Gauss-Seidel Method . . . . .	125
5.2.5	The Householder Transformation . . . . .	127
5.3	The Eigenvalue Problem . . . . .	130
5.3.1	Coupled Oscillators . . . . .	130
5.3.2	Basic Properties . . . . .	131
5.3.2.1	The Power Method for Finding Eigenvalues . . . . .	132
5.3.2.2	The Inverse Power Method . . . . .	133

5.3.3	Tridiagonal Symmetric Matrices . . . . .	134
5.3.4	The Role of Orthogonal Matrices . . . . .	138
5.3.5	The Householder Method for Eigenvalues . . . . .	139
5.3.6	The Lanczos Algorithm . . . . .	139
6.	Exercises in Monte Carlo . . . . .	147
6.1	The Potential Energy of the Oxygen Atom . . . . .	147
6.2	Oxygen Potential Energy with Metropolis . . . . .	151
6.3	Radiation Transport . . . . .	153
6.4	An Inverse Problem with Monte Carlo . . . . .	157
7.	Finite Element Methods . . . . .	161
7.1	Basis Functions – One Dimension . . . . .	162
7.2	Establishing the System Matrix . . . . .	164
7.2.1	Model Problem . . . . .	165
7.2.2	The “Classical” Procedure . . . . .	165
7.2.3	The Galerkin Method . . . . .	166
7.2.4	The Variational Method . . . . .	168
7.3	Example One-dimensional Program . . . . .	169
7.4	Assembly by Elements . . . . .	171
7.5	Problems in Two Dimensions . . . . .	172
7.5.1	Element Functions . . . . .	172
7.5.2	Laplace’s Equation . . . . .	174
8.	Digital Signal Processing . . . . .	181
8.1	Fundamental Concepts . . . . .	181
8.2	Sampling: Nyquist Theorem . . . . .	181
8.3	The Fast Fourier Transform . . . . .	184
8.4	Phase Problems . . . . .	189
9.	Chaos . . . . .	193
9.1	Functional Iteration . . . . .	193
9.2	Finding the Critical Values . . . . .	202
10.	The Schrödinger Equation . . . . .	209
10.1	Removal of the Time Dependence . . . . .	209
10.2	Reduction of the Two-body System . . . . .	210
10.3	Expansion in Partial Waves . . . . .	211
10.4	The Scattering Problem . . . . .	213
10.4.1	The Scattering Amplitude . . . . .	215
10.4.2	Model Nucleon-nucleon Potentials . . . . .	223
10.4.3	The Off-shell Amplitude . . . . .	225
10.4.4	A Relativistic Generalization . . . . .	231
10.4.5	Formal Scattering Theory . . . . .	232
10.4.6	Modeling the t-matrix . . . . .	233
10.4.7	Solutions with Exponential Potentials . . . . .	235
10.4.8	Matching with Coulomb Waves . . . . .	239
10.5	Bound States of the Schrödinger Equation . . . . .	242
10.5.1	Nuclear Systems . . . . .	244
10.5.2	Physics of Bound States: The Shell Model . . . . .	244
10.5.3	Hypernuclei . . . . .	247

10.5.4	The Deuteron . . . . .	248
10.5.5	The One-Pion-Exchange Potential . . . . .	251
10.6	Properties of the Clebsch-Gordan Coefficients . . . . .	256
10.7	Time Dependent Schrödinger Equation . . . . .	258
11.	The N-body Ground State . . . . .	269
11.1	The Variational Principle . . . . .	270
11.1.1	A Sample Variational Problem . . . . .	271
11.1.2	Variational Ground State of the $^4\text{He}$ Nucleus . . . . .	274
11.1.3	Variational Liquid $^4\text{He}$ . . . . .	278
11.2	Monte Carlo Green's Function Methods . . . . .	281
11.2.1	The Green's Function Approach . . . . .	282
11.2.2	Choosing Walkers for MCGF . . . . .	289
11.3	Alternate Energy Estimators . . . . .	290
11.3.1	Importance Sampling . . . . .	292
11.3.2	An Example Algorithm . . . . .	294
11.4	Scattering in the N-body System . . . . .	295
11.5	More General Methods . . . . .	299
12.	Divergent Series . . . . .	303
12.1	Some Classic Examples . . . . .	303
12.2	Generalizations of Cesàro Summation . . . . .	305
12.3	Borel Summation . . . . .	307
12.3.1	Borel's Differential Form . . . . .	307
12.3.2	Borel's Integral Form . . . . .	309
12.4	Padé Approximants . . . . .	311
13.	Scattering in the N-body System . . . . .	319
13.1	Single Scattering . . . . .	319
13.2	First Order Optical Potential . . . . .	322
13.2.1	Calculating the Non-local Potential . . . . .	324
13.2.2	Solving with a Non-local Potential . . . . .	328
13.3	Double Scattering . . . . .	331
13.3.1	Relation of Double Scattering to Coherence . . . . .	335
13.4	Scattering from Fixed Centers . . . . .	336
13.5	The Watson Multiple Scattering Series . . . . .	340
13.6	The KMT Optical Model . . . . .	346
13.7	Medium Corrections . . . . .	346
Appendix A	Programs . . . . .	355
A.1	Legendre Polynomials . . . . .	355
A.2	Gaussian Integration . . . . .	356
A.3	Spherical Bessel Functions . . . . .	359
A.4	Random Number Generator . . . . .	362
<i>Index</i>		363