

STOCHASTIC
MODELLING
AND APPLIED
PROBABILITY

21

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Stochastic Integration and Differential Equations

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