

$N(dsdu, \omega)$

Kiyosi Itô

# Stochastic Processes

LECTURES GIVEN AT AARHUS UNIVERSITY

Edited by  
Ole E. Barndorff-Nielsen  
and Ken-iti Sato



Springer

# Table of Contents

<b>0 Preliminaries .....</b>	<b>1</b>
0.1 Independence .....	1
0.2 Central Values and Dispersions.....	5
0.3 Centralized Sum of Independent Random Variables .....	12
0.4 Infinitely Divisible Distributions .....	18
0.5 Continuity and Discontinuity of Infinitely Divisible Distributions .....	25
0.6 Conditional Probability and Expectation .....	27
0.7 Martingales .....	32
<b>1 Additive Processes</b>	
<b>(Processes with Independent Increments).....</b>	<b>39</b>
1.1 Definitions .....	39
1.2 Decomposition of Additive Processes .....	41
1.3 The Lévy Modification of Additive Processes Continuous in Probability .....	45
1.4 Elementary Lévy Processes .....	50
1.5 Fundamental Lemma .....	58
1.6 Structure of Sample Functions of Lévy Processes (a) .....	61
1.7 Structure of Sample Functions of Lévy Processes (b) .....	68
1.8 Three Components of Lévy Processes .....	74
1.9 Random Point Measures .....	77
1.10 Homogeneous Additive Processes and Homogeneous Lévy Processes .....	83
1.11 Lévy Processes with Increasing Paths .....	85
1.12 Stable Processes .....	88
<b>2 Markov Processes .....</b>	<b>93</b>
2.1 Transition Probabilities and Transition Operators on Compact Metrizable Spaces .....	93
2.2 Summary of the Hille–Yosida Theory of Semi-Groups .....	95
2.3 Transition Semi-Group .....	101
2.4 Probability Law of the Path .....	103

## XII Table of Contents

2.5	Markov Property . . . . .	110
2.6	The $\sigma$ -Algebras $\bar{\mathcal{B}}$ , $\bar{\mathcal{B}}_t$ , and $\overline{\mathcal{B}(S)}$ . . . . .	115
2.7	Strong Markov Property . . . . .	118
2.8	Superposition of Stopping Times . . . . .	123
2.9	An Inequality of Kolmogorov Type and its Application . . . . .	124
2.10	Hitting Times of Closed Sets . . . . .	129
2.11	Dynkin's Formula . . . . .	130
2.12	Markov Processes in Generalized Sense . . . . .	135
2.13	Examples . . . . .	143
2.14	Markov Processes with a Countable State Space . . . . .	148
2.15	Fine Topology . . . . .	154
2.16	Generator in Generalized Sense . . . . .	159
2.17	The Kac Semi-Group and its Application to the Arcsine Law . . . . .	164
2.18	Markov Processes and Potential Theory . . . . .	170
2.19	Brownian Motion and the Dirichlet Problem . . . . .	172
<b>Exercises</b> . . . . .		179
E.0	Chapter 0 . . . . .	179
E.1	Chapter 1 . . . . .	188
E.2	Chapter 2 . . . . .	190
<b>Appendix: Solutions of Exercises</b> . . . . .		193
A.0	Chapter 0 . . . . .	193
A.1	Chapter 1 . . . . .	215
A.2	Chapter 2 . . . . .	221
<b>Index</b> . . . . .		231