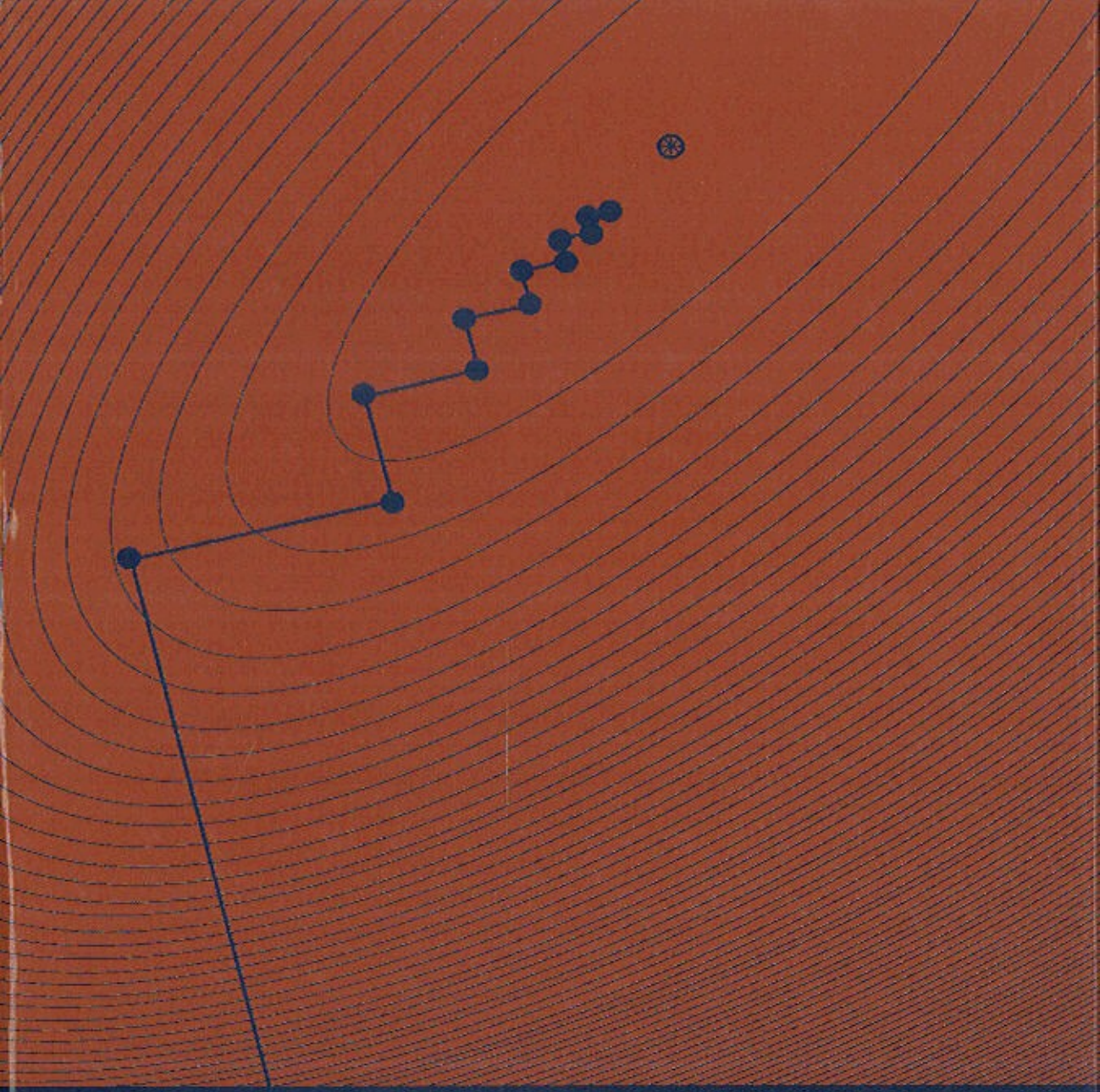

Applied Optimization

Formulation and Algorithms for Engineering Systems

Ross Baldick

CAMBRIDGE

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