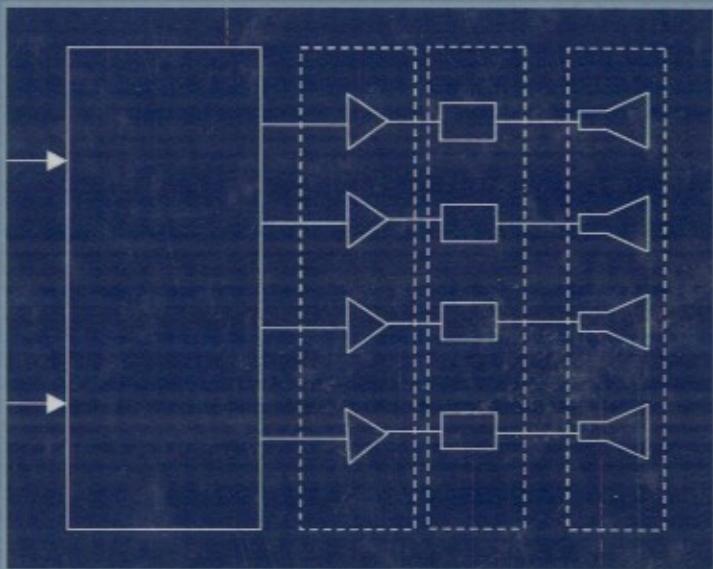


# PHASED ARRAY ANTENNAS

Floquet Analysis, Synthesis,  
BFNs, and Active Array Systems



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