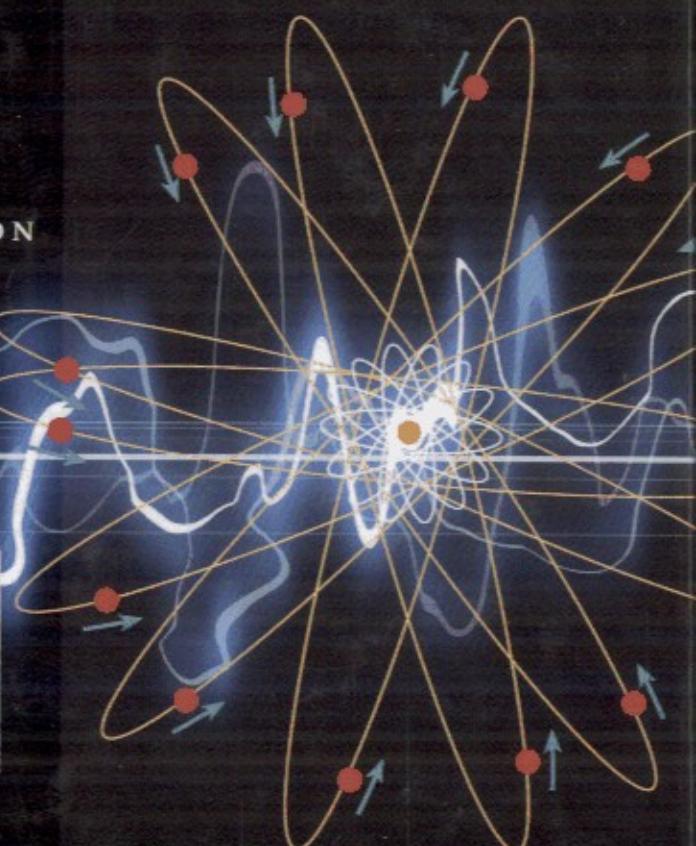
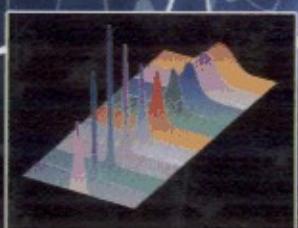


# Ultrashort Laser Pulse Phenomena

*Fundamentals, Techniques,  
and Applications on a  
Femtosecond Time Scale*

SECOND EDITION



Jean-Claude Diels  
Wolfgang Rudolph

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