

ELEMENTS OF

STATISTICAL MECHANICS

**With an Introduction to
Quantum Field Theory and
Numerical Simulation**

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