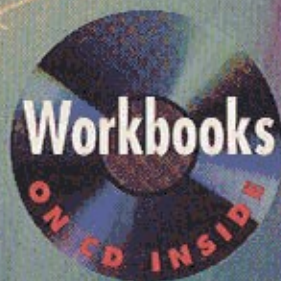


PHYSICAL CHEMISTRY

Quantum Mechanics

$$\hat{H}\Psi = E\Psi$$

HORIA METIU



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