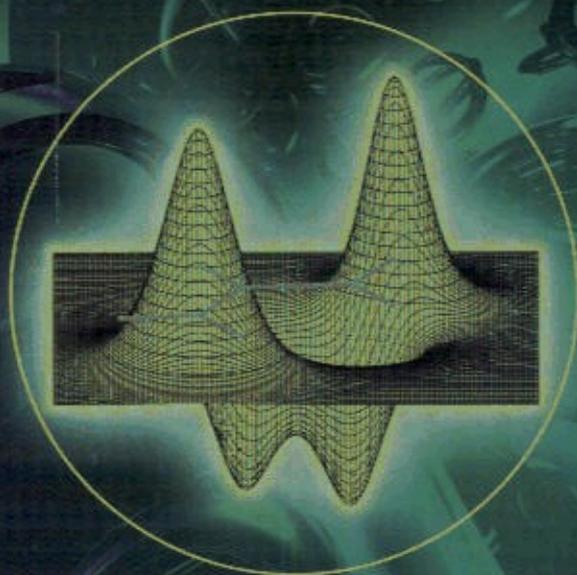


Second Edition

# Introduction to Computational Chemistry



Frank Jensen

 WILEY

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