

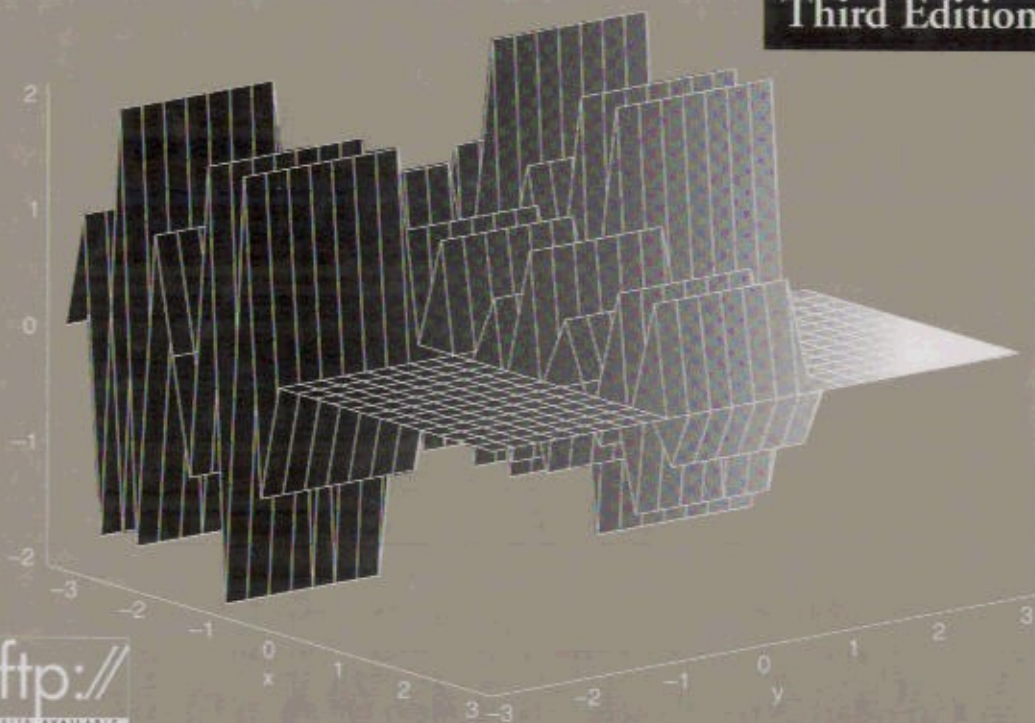
*Pure and Applied Mathematics:
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Partial Differential Equations of Applied Mathematics

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