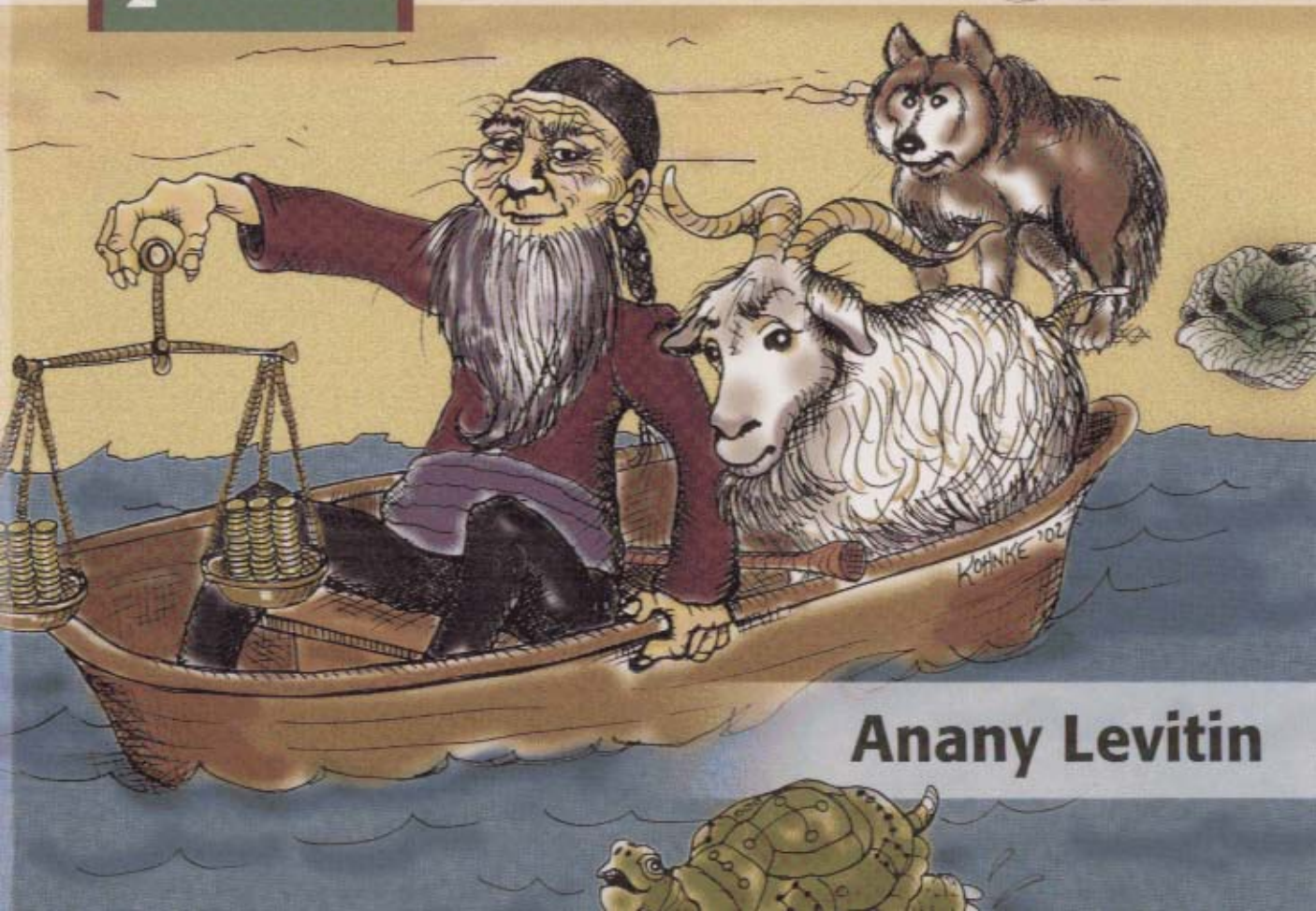
Introduction to The Design & Analysis of Algorithms

2ND EDITION



Anany Levitin

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