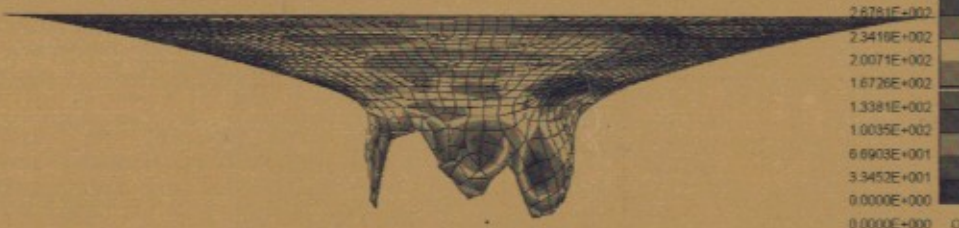


Finite Element Analysis

Thermomechanics of Solids

PLATE 104
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