

STOCHASTIC
MODELLING
AND APPLIED
PROBABILITY

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25

Controlled Markov Processes and Viscosity Solutions

Second Edition



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