

# BUCKLING EXPERIMENTS

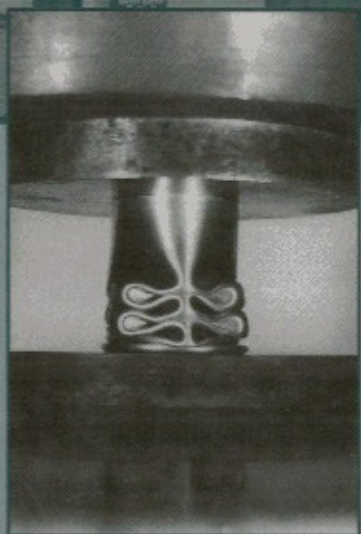
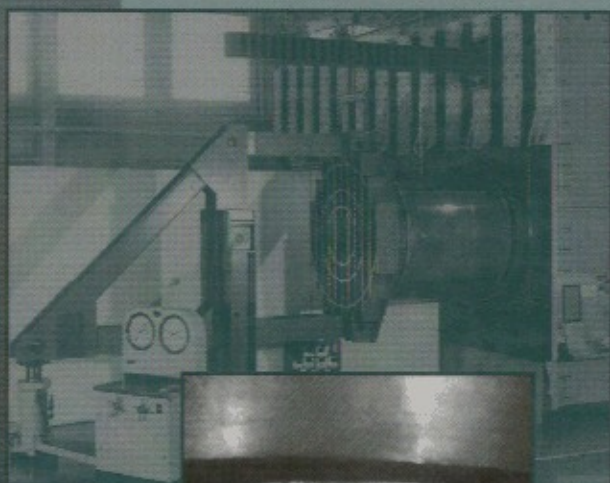
Experimental Methods in Buckling  
of Thin-Walled Structures

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**Volume 1**

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Basic  
Concepts,  
Columns,  
Beams,  
and Plates



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