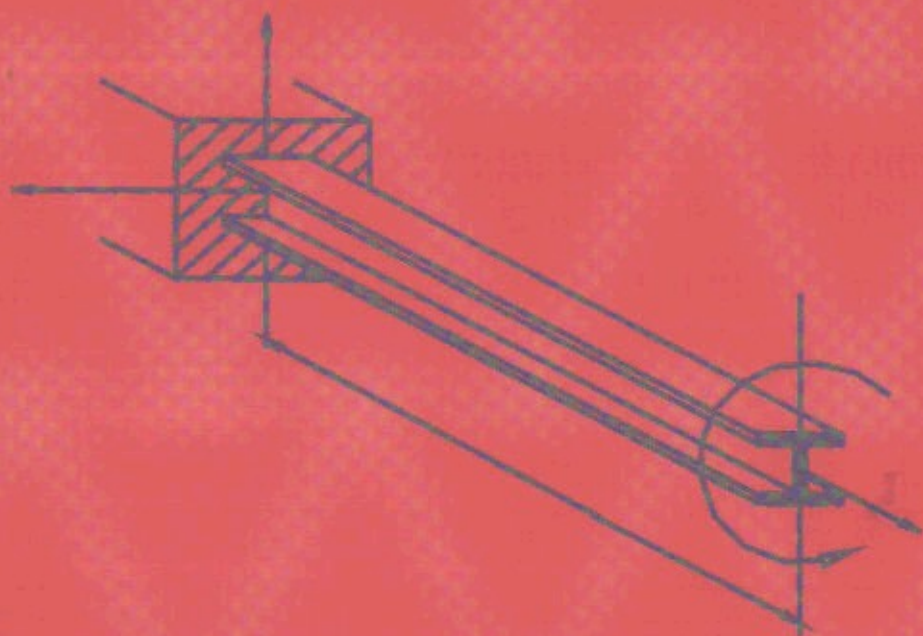


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Stability Analysis and Design of Structures



 Springer

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