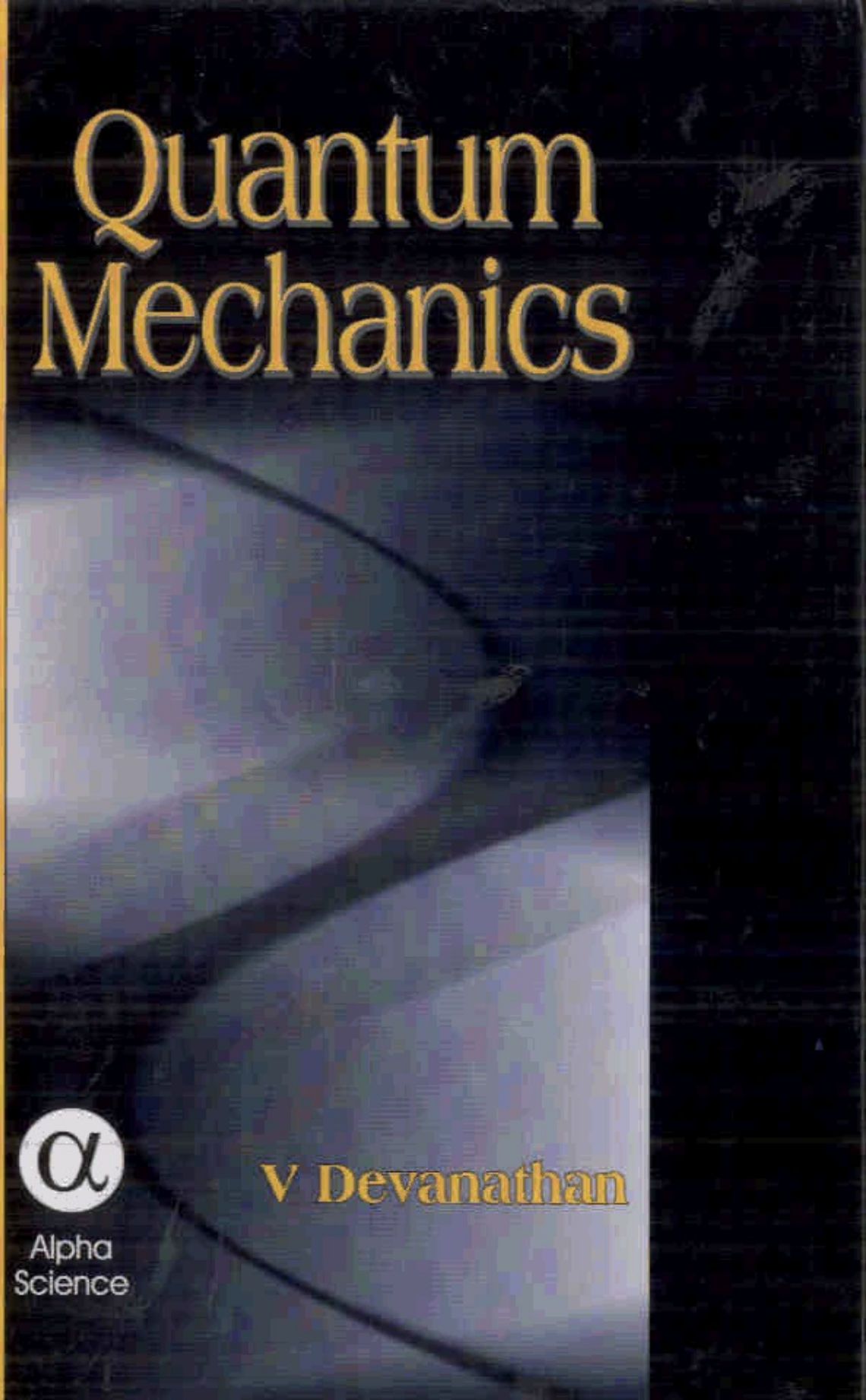
Quantum Mechanics



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Alpha Science

Contents

Preface *vii*

1 Foundations of Quantum Mechanics **1**
1.1 Wave-particle duality 1
1.2 Wave packets 2
1.3 Fourier transforms 6
1.4 The Schrödinger Equation 7
1.4.1 Physical interpretation of wave function 10
1.4.2 Expectation values of dynamical variables 11
1.4.3 Dirac's bra and ket vectors 11
1.5 The continuity equation 12
1.6 Ehrenfest's theorem 13
1.7 Heisenberg's uncertainty relation 15
1.8 Postulates of quantum mechanics 19
Review Questions 20
Problems 21
Solutions to Problems 22

2 Bound States and Potential Barriers **27**
2.1 One-dimensional square well 27
2.1.1 Infinite square well 27
2.1.2 Finite square well (Type A) 29
2.1.3 Finite square well (Type B) 32
2.2 Three-dimensional square well 34
2.2.1 Interior Region - Region I 36
2.2.2 Exterior region - Region II 37
2.2.3 Energy eigenvalues 38
2.3 The linear harmonic oscillator 39
2.4 The three-dimensional harmonic oscillator 43
2.4.1 In Cartesian coordinates 43
2.4.2 In spherical coordinates 44
2.5 The rigid rotator 50

2.6 The hydrogen atom . . . 52
2.7 One-dimensional potential barrier . . . 61
2.8 Three-dimensional potential barrier . . . 65
Review Questions . . . 66
Problems . . . 67
Solutions to Problems . . . 68

3 The WKB Approximation **78**
3.1 The WKB wave functions . . . 78
3.2 The connection formulae . . . 82
3.3 Application to bound states . . . 86
3.4 Transmission through a potential barrier . . . 89
Review Questions . . . 92
Problems . . . 92
Solutions to Problems . . . 92

4 Approximate Methods for Bound States **98**
4.1 Time-independent perturbation theory . . . 98
4.1.1 Non-degenerate unperturbed states . . . 100
4.1.2 Degenerate unperturbed states . . . 106
4.2 The variational method . . . 112
4.3 Helium atom . . . 115
4.3.1 Using perturbation method . . . 116
4.3.2 Using variational method . . . 119
4.4 Muonic atom . . . 121
4.5 The hydrogen molecule . . . 123
Review Questions . . . 132
Problems . . . 133
Solutions to Problems . . . 134

5 Angular Momentum, Spin and Isospin **139**
5.1 Quantum mechanical definition of angular momentum . . . 139
5.2 Ladder operators . . . 141
5.3 Allowed values of j and m . . . 142
5.4 Matrix elements of angular momentum operators . . . 143
5.5 Construction of angular momentum matrices . . . 145
5.6 Angular momentum state vectors . . . 146
5.7 System of two spin-$\frac{1}{2}$ particles . . . 147
5.8 Coupling of two angular momenta . . . 148
5.9 Isospin formalism . . . 151

5.9.1 Isospin of two-nucleon system 152
5.9.2 Isospin of pions and pion-nucleon system 152
Review Questions 155
Problems 155
Solutions to Problems 156

6 Equations of Motion and Matrix Mechanics **162**
6.1 The Schrödinger picture 163
6.2 The Heisenberg picture 164
6.3 The Interaction picture 166
6.4 The Linear Harmonic Oscillator 168
6.4.1 Solution using the Schrödinger Picture 168
6.4.2 Solution using the Heisenberg Picture (Matrix mechanics) . . 170
6.4.3 Method of Second Quantization 173
Review Questions 178
Problems 178
Solutions to Problems 179

7 Time-Dependent Perturbation Theory **183**
7.1 The interaction picture 183
7.2 First order perturbation 186
7.3 Constant perturbation 188
7.4 Transition to the continuum 189
7.5 Elastic scattering 190
7.6 Harmonic perturbation 192
7.7 Adiabatic and Sudden approximations 192
Review Questions 197
Problems 197
Solutions to Problems 198

8 Interaction of Radiation with Atom **201**
8.1 The semi-classical treatment 201
8.2 Selection rules for electric dipole transition 205
8.3 Spontaneous emission 206
8.4 Induced or stimulated emission 207
8.5 The Einstein coefficients 208
8.5.1 Masers and Lasers 210
Review Questions 211
Problems 211
Solutions to Problems 212

9 Theory of Scattering **216**
9.1 The Green's function technique 217
9.2 The Born approximation 219
9.2.1 Rutherford scattering formula 220
9.3 The method of partial waves 221
9.3.1 Partial wave analysis 221
9.3.2 External and internal logarithmic derivatives 225
9.3.3 The square well potential 226
9.3.4 Scattering cross section and the optical theorem 226
9.3.5 The complex potential 229
9.4 Very low energy scattering 230
9.4.1 The scattering length 230
9.4.2 The effective range theory 232
9.4.3 Low energy neutron-proton scattering 234
Review Questions 235
Problems 236
Solutions to Problems 237

10 Transformation between Centre-of-Momentum and Laboratory Systems **245**
10.1 Non-relativistic treatment 245
10.2 Relativistic treatment 249
10.2.1 Transformation of velocities, momenta and angles 249
10.2.2 Transformation of differential cross section 251
10.3 Relativistic kinematics 258
10.4 Transformation laws - a consistency check 260
Review Questions 263
Problems 264
Solutions to Problems 264

11 Relativistic Wave Equations **270**
11.1 The Klein-Gordon Equation 270
11.1.1 Pionic atoms and pion-nucleus scattering 272
11.2 The Dirac Equation 274
11.2.1 The free particle solutions 278
11.2.2 The hole theory 281
11.3 Properties of the Dirac electron 283
11.3.1 Spin of the Dirac electron 283
11.3.2 Magnetic dipole moment of the electron 285
11.3.3 The velocity operator 286
11.3.4 Average or expectation value of the velocity 287

11.4 Relativistic invariance of Dirac equation 290
11.4.1 Lorentz transformation operator 292
11.4.2 Demonstration of the relativistic invariance 295
11.5 Parity, charge conjugation and time reversal operations 298
11.5.1 The parity operation 298
11.5.2 Charge conjugation 300
11.5.3 Time reversal operation 304
11.6 Feynman's theory of positrons 306
Review Questions 309
Problems 309
Solutions to Problems 310

Appendix A
Units and Dimensions 314

Appendix B
Tables of Clebsch-Gordon Coefficients 321

Appendix C
Vectors and Tensors in the Spherical Basis 322

Appendix D
The Spherical Harmonics 325

Appendix E
The Density of Final States 327

Subject Index **330**