

# COSMOLOGICAL RELATIVITY

The Special and General Theories  
for the Structure of the Universe



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