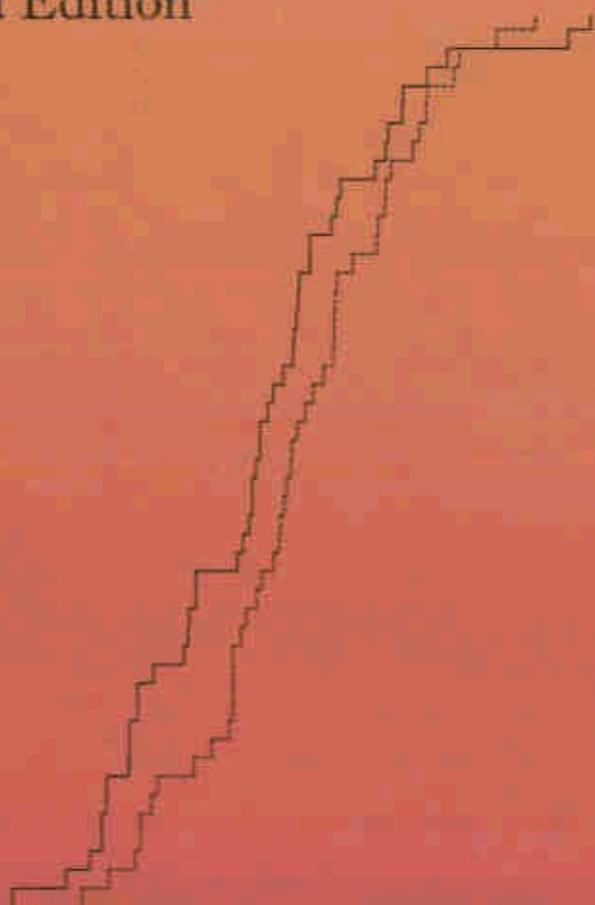


Frederick James

Statistical Methods in Experimental Physics

2nd Edition



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